

Risk Factors and Distributions from Long-Short Trading Strategies: A Monte Carlo Approach

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Abstract

We propose a simulation-based methodology for evaluating the significance of returns to long-short trading strategies and for investigating the effects of various risk adjustments and other return characteristics. As an example, we apply our approach to three widely studied long-short trading strategies: momentum, value and size. We generate empirical distributions for various moments of strategy returns and examine the distributions of Sharpe ratios, skewness and kurtosis, as well as factor-model alphas and betas. We find that if investors demand a premium for holding portfolios with high skewness/kurtosis, a significant fraction of long-short trading profits can be explained.

1. Introduction

A cornerstone of financial economics is the risk-reward relationship, which implies that an investor can only achieve higher expected returns by taking on more priced risk. Investment strategies that appear to consistently produce returns that are greater than the returns required to compensate for risk are therefore deemed “puzzles”.

Three of the most widely studied risk-return puzzles are those related to size, value and momentum. Over the past 80 years returns to investing in small firms have been higher than returns on large-cap stocks, a finding known as the “size puzzle”. Similarly, stocks with high book-to-market ratios (aka value stocks) seem to outperform stocks with low book-to-market ratios (aka growth stocks), a finding called the “value puzzle”. Stocks with high prior returns (aka winners) tend to outperform stocks with poor recent performance (aka losers), a finding known as the “momentum puzzle”. Investors who took long positions in the higher-returning portfolio and shorted its lower-returning counterpart (e.g., buy winners and sell losers – the momentum strategy) thus earned considerable profit. Such profits even seem to hold when standard adjustments are made to control for risks associated with the applicable trading strategies.¹

Key research questions when studying such long-short investment strategies include: whether risk has been appropriately captured in the investigations, whether excess risk-adjusted returns are statistically significant or random chance, and if the apparently anomalous returns to the long-short trading strategies can be explained by various interesting risk

¹The “size effect” has been documented by Banz (1981) and Reinganum (1981), among others, with numerous subsequent studies (e.g. Fama and French, 1992) showing that the CAPM fails to explain the size effect. The “value puzzle” has been studied by Stattman (1980), Basu (1983), Rosenberg, Reid, and Lanstein (1985), and Fama and French (1992), among many others. As for momentum, Jegadeesh and Titman (1993) find that a portfolio that buys stocks with high recent returns and sells stocks with low recent returns produces significant profits over the period of 3 to 12 months. This finding of momentum in stock returns has been confirmed by numerous authors in both U.S. and international data. Neither the CAPM, nor the 3-factor model of Fama and French (1993) in conditional or unconditional form, can explain momentum profits, although profits from some other strategies do tend to fade as more factors are added, as discussed in Fama and French (1996).

factors. In this paper we propose a simulation-based methodology for investigating such issues.

We begin by computer-generating millions of bootstrapped portfolio-return simulations in order to produce empirical distributions of raw and risk-adjusted profits to trading strategies that mimic size, value or momentum investing; except that, in our simulated portfolios, stocks selected for long or short positions are selected randomly (which is appropriate under the null hypothesis of no strategy effect) rather than being selected based on their past performance or characteristics. The resulting simulated portfolio return distributions are thus produced under the null hypothesis of no strategy effect. Against these null distributions we compare actual market-observed profits, risk and risk-adjusted returns, from the true size, value and momentum investment strategies so that we can draw conclusions about the statistical significance of the various trading strategies and the sources of supposedly anomalous excess returns.² While the methodology of simulating null distributions for use in statistical investigations has been employed in other areas of finance,³ we believe this is the first paper to use this particular approach to study portfolio-based trading strategies including momentum, value and size effects.⁴

²For example, consider a typical momentum strategy which, at the beginning of every month, calls for buying stocks with high prior returns, and shorting stocks with low prior returns, to form a zero-cost investment portfolio that is then held for several subsequent months. A “random momentum strategy” simply calls for buying some randomly selected stocks and shorting other randomly selected stocks and holding the resulting zero-cost random portfolio. The random strategy mimics the true momentum strategy except that, rather than buying recent winners and shorting recent losers, the random strategy selects stocks randomly. To determine the significance of returns from the true momentum strategy, we can therefore run a large number of random long-short simulations to obtain an empirical distribution of returns to such a random momentum strategy; i.e., we obtain the return distribution under the null hypothesis of no strategy effect. If the market-observed momentum-return is an outlier against the empirically-generated null distribution, then we can confidently conclude that momentum profits are indeed abnormally high. If, on the other hand, the market-observed momentum return falls in the body of the null distribution, then the market-observed value cannot be considered abnormal. Similar techniques can also be used to address more advanced and interesting questions including identifying the sources of whatever abnormalities or “excess” returns may be found, including the effects of various basic and complex risk adjustments, as explained below in this paper.

³See, e.g., Donaldson, Kamstra and Kramer (2008).

⁴For example, our approach differs from that in an interesting paper by Conrad et al. (2003), who investigate the effects of data snooping biases on the finding that future stock returns are related to certain characteristics of firms. Conrad et al. (2003) assign stocks into portfolios based on randomly generated

To undertake our investigation, we use bootstrap simulation methods to generate empirical distributions for the mean, standard deviation, skewness and kurtosis of monthly returns of the momentum, value and size strategies. We also examine the distributions of Sharpe ratios, ratios of average returns to skewness and kurtosis, as well as CAPM and 3- and 4-factor alphas and betas. We evaluate the significance of raw and risk-adjusted profits to the size, value and momentum investment strategies in the entire 1927-2006 sample, as well as in subperiods, and investigate effects of various factors on returns.

Our results reveal that the average raw returns from the size, value and momentum strategies are significantly larger than any random draw from our simulations, thereby confirming that raw profits from momentum, value and size investing are indeed unusually high. However, the standard deviations of returns from all of the three trading strategies (size, value and momentum) are also unusually high. Skewness and kurtosis of all the trading strategies also fall in the tails of the empirical distributions, but their magnitudes are not entirely uncommon since a sizeable fraction of random portfolios have as extreme values of the third and fourth moments.

We consider several methods of risk adjustment. First, we examine the ability of the market model to explain the profits from the three strategies. We show that the market model does indeed help to explain a fraction of profits from value and size investing, but conversely magnifies momentum profits (which is itself a puzzling result). We then consider multifactor models that do not include own-factors specifically designed to explain the puzzle

firm characteristics and conclude that data snooping can account for roughly 50 percent of the in-sample relationship between firm characteristics and future returns. Our approach randomizes directly at the stock selection level, rather than the characteristic level, and thus more closely mimics portfolio formation of common investment strategies and is more directly tied to the null hypothesis. This allows us to generate empirical distributions of raw and risk-adjusted returns, as well as distributions of higher moments of returns of such random portfolios, and compare true values of mean and other moments of returns from different investment strategies against such empirical distributions and evaluate their significance. This produces results that in some cases are contrary to Conrad et al. (2003); for example we provide direct evidence that raw returns to size, value, and momentum investing are indeed highly abnormal, as they fall far outside the relevant empirical distributions, and that the profitability of these investment strategies is related to skewness and kurtosis.

(such as HML factor for value investing), and find that such “Fama-French factors” do no better than the market model in explaining size and value profits, and do considerably worse at explaining momentum returns. Finally, we consider 3-factor risk adjustments for size and value strategies and 4-factors for momentum and find that, only when own-factors are included in the models, do the trading profits cease to appear unusual (as one would expect given the construction of the factors).

We also find that for each of the three strategies under consideration (size, value and momentum) the Sharpe ratio, as well as the ratio of average returns to kurtosis, fall in the far right tail of the corresponding empirical distributions, whereas the ratio of average returns to skewness falls close to the center. These findings suggest that the high profits from momentum, value, and size investing relate in part to volatility, skewness, and kurtosis of their returns, with the most likely explanation being a return compensating for skewness. If investors demand a high premium for holding portfolios with high skewness, for example, at least some fraction of profits from momentum, value and size investing can be explained away.⁵

Our results reveal that the profitability of the momentum, value, and size trading strategies may be attributable at least in part to risk associated with skewness and kurtosis. We therefore construct several skewness and kurtosis factors and assess their ability to aid in the explanation of profits from the three investment strategies. We demonstrate below that these factors can indeed help to explain trading profits and variation in profits. The loadings on

⁵The notion that investors could have preferences over skewness is supported by the literature. For example, Kraus and Litzenberger (1976) develop an asset pricing model in which, in addition to preferences over the first two moments, investors exhibit preferences over skewness. Harvey and Siddique (2000) test the implication of a three-moment CAPM that investors demand a risk premium for stocks that have high negative coskewness with the market. The coskewness factor they construct earns 3.6% annually and helps explain a portion of momentum profits. Smith (2007) finds that coskewness is an important determinant of the returns to equity; he shows that, unlike the CAPM and Fama-French three-factor model, a model that includes a coskewness factor is not rejected by the data. Chung et al. (2006) explore a set of return co-moments of order 3 to 10, and find that the commonly used Fama-French three factors in part proxy for a set of higher-order co-moments.

the skewness and kurtosis factors we construct are highly significant and inclusion of these factors greatly improves the proportion of variability of investment profits explained. In the case of the value strategy, for example, inclusion of even our simplest skewness and kurtosis factors eliminates the significance of trading strategy profits.

The rest of the paper proceeds as follows. Section 2 provides a brief discussion of our data construction and the three popular asset pricing anomalies (size, value and momentum) that we use as test-cases for our simulation methodology. Section 3 describes our methodology. Our empirical results from simulated distributions are reported in Section 4. In Section 5 we then construct skewness and kurtosis factors and examine their ability to explain profits from the momentum, value and size strategies. Section 6 concludes.

2. Data and Puzzles

Recall that the size trading strategy calls for taking long positions in small-cap stocks and short positions in large-cap stocks. Similarly, the value trading strategy calls for shorting stocks with low book-to-market ratios and going long in stocks with high book-to-market ratios.

To construct the long and short positions of the size and value portfolios, we follow Davis, Fama, and French (2000). At the end of June for each year t from 1926 to 2006, we assign stocks into two groups – B (big) and S (small) – depending on whether their market capitalization is above or below the median of all NYSE stocks at that time. Independently, stocks are also assigned into three groups – H (high), M (medium) and L (low) – depending on whether their book-to-market ratio falls in the top 30, middle 40, or bottom 30 percent of book-to-market values of all NYSE stocks at that time. Book and market equity are measured as in Davis, Fama, and French (2000). Value-weighted returns from July of year t to June of year $t+1$ are then obtained for each of the six portfolios formed as the intersection

of the two size and three book-to-market groups (which, following tradition, are noted: BH, BM, BL, SH, SM, SL).

Tradition in this literature (e.g., Davis, Fama, and French (2000)), defines the size strategy as taking each July a \$1 long position in the portfolio of small stocks $[SH + SM + SL]/3$ and a \$1 short position in the portfolio of big stocks $[BH + BM + BL]/3$. Similarly, a value strategy involves taking each July a \$1 long position in the portfolio of high book-to-market “value” stocks $[BH + SH]/2$, and a \$1 dollar short position in the portfolio of low book-to-market “growth” stocks $[BL + SL]/2$. We include all common stocks available on the CRSP and COMPUSTAT databases with non-negative book-to-market values, as well as stocks with historical book-to-market values from Ken French’s data library.⁶

Next, recall that the momentum strategy shorts stocks that have performed poorly in the past (aka, losers) and buys stocks that have performed well in the past (aka, winners). To construct the momentum portfolio, we mimic the details outlined on Ken French’s website. In particular, on the last trading day of each calendar month $t - 1$ from June 1926 to October 2006, we compute compounded returns from $t - 12$ to $t - 2$ for each common stock available on the CRSP database. We require a valid return in month $t - 2$ and returns over the entire 11 months, a price as of the end of $t - 13$, and positive market capitalization (i.e., size) as of the end of $t - 1$. We assign stocks into three portfolios based on this prior return (called: High Runup, Medium Runup and Low Runup), using the 30th and 70th percentiles as breakpoints. We independently assign stocks into two portfolios (called Big and Small) depending on whether $t - 1$ market size is above or below the median NYSE size. Following tradition, we then form six value-weighted portfolios from the intersection. The winner return is the month t return on the portfolio $[Small\ with\ High\ Runup + Big\ with\ High\ Runup]/2$, while the loser return is the return from the portfolio $[Small\ with\ Low\ Runup + Big\ with$

⁶http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Low Runup]/2. The difference between the returns on these two portfolios – noted WL, for “winners minus losers” – represents momentum gain.⁷

— **Table 1 Goes Here** —

The average returns to size, value, and momentum strategies produced in our data are comparable to those documented in the previous literature, and are reported in Table 1. All strategies earned on average positive returns over the entire 80-year period: winners outperformed losers by 0.77% per month; high book-to-market stocks beat low book-to-market stocks by 0.46% monthly; and smaller stocks topped bigger stocks by 0.24% per month. Momentum and value strategies have been consistent, producing large and rather stable profits throughout different periods. Size, on the other hand, has been the least reliable strategy; it was unprofitable in the 1947-1966 period and produced a negligible return starting in the 1980s.

3. Methodology

In this section we outline a methodology for evaluating the returns and risks of the investment strategies. The methodology is based on generating empirical “null distributions” of the raw and risk-adjusted return measures for portfolios comprised of stocks that are randomly selected (rather than strategically selected) in keeping with the null hypotheses being examined in each of the empirical tests. The returns and risk of the true investment strategy are then compared to the empirical null distributions, which allows us to draw conclusions

⁷To check the robustness of our findings, we examined other potential momentum strategies. In particular, we explored the strategy the buys the decile of stocks with the highest prior six-month return, sells a decile of stocks with the lowest six-month return, and holds the position for the following six months. We also considered skipping a month between formation and holding periods. The results obtained when exploring these alternative strategies are similar to those presented in the paper, thereby lending confidence to the robustness of our findings.

about the significance of the trading strategy statistics. We now describe the details of the methodology more carefully by considering the momentum strategy as an example.

To form random “null hypothesis” portfolios, we use the same universe of stocks that are eligible to be considered for inclusion in the true momentum portfolio. In every month we therefore take a long position in a portfolio of randomly chosen stocks (the pseudo W “winners” portfolio) and a short position in a portfolio of different but also randomly selected stocks (the pseudo L “losers” portfolio), and hold the resulting pseudo WL (winners minus losers) position during month t .⁸ The random strategy thus follows the true momentum strategy precisely, except that every month stocks are selected into winner and loser portfolios randomly instead of according to their actual returns; i.e., the pseudo WL portfolio contains randomly selected stocks, not stocks that are necessarily true winners and true losers.⁹

For each simulation sequence described above, we thus obtain a time series of monthly returns of pseudo-winners, pseudo-losers and pseudo-winners-minus-losers in this ‘random momentum’ strategy for the years 1927-2006. We calculate the mean, standard deviation, skewness, and kurtosis of the monthly returns from this time series, as well as ratios of average returns to standard deviation, skewness, and kurtosis. We also estimate alphas and loadings (betas) from market and multifactor models described below. We repeat the entire procedure described above 10,000 times to generate 10,000 simulation sequences for the years 1927-2006. These 10,000 sequences yield the empirical distributions of raw and risk-adjusted returns, as well as their risk measures, under the null hypothesis that the true momentum portfolio does not behave any differently than randomly selected portfolios.

⁸Since the true momentum strategy we examine calls for taking, at the beginning of month t , a position in stocks with the highest and lowest returns during the $t - 12$ to $t - 2$ period, we do not include stocks with missing returns during this 11-month period in the pool of stocks from which we draw randomly; we also exclude stocks that are missing either market capitalization data as of the end of $t - 1$ or price data as of the end of month $t - 13$.

⁹Note the difference between our randomization approach and that of Conrad et al. (2003), as explained in footnote 4 above.

To generate the empirical distributions for size and value strategies, we again closely follow the true strategies but instead select stocks into small, big, value, and growth portfolios randomly out of the universe of stocks that have the necessary data to be considered for inclusion into the true portfolios.¹⁰

We use the resulting empirical distributions to evaluate the performance of the momentum, value and size strategies. We examine where the value of each parameter in question (e.g., average return or alpha) of the true strategies falls in the empirical distribution of the corresponding parameters from the random strategies (in other words, does the true application of the strategy produce returns that look different than returns one would find by randomly selecting portfolios). If the true value falls in the body of the null distribution, we can argue that little is puzzling about such value. On the other hand, if it falls in the tails of the distribution, and is thus an outlier, we can conclude that such value is indeed unusually high or low.

As an illustration, consider determining whether the average returns of the momentum strategy are statistically different from zero at the 10% significance level in a two-tailed test. The traditional methods call for calculating a statistic such as a p-value and comparing it to a critical value. We propose using the empirical distribution of mean returns from the random strategies to estimate the p-value empirically. If the value of the average return of the true strategy falls either above the 95th percentile or below the 5th percentile of the empirical distribution, we could conclude that the value of this parameter is indeed puzzling at the 10% significance level. Otherwise, if it falls between the 5th and 95th percentiles, we can argue that its value is not significantly different from the returns we would have obtained by investing in random portfolios and is therefore not unusual.

¹⁰Obtaining raw returns of the long, short, and long minus short sides of the random momentum, value, and size strategies takes approximately three weeks time running three simultaneous jobs on the powerful WRDS server. Additional computations, such as estimating alphas and loadings for all random draws and different subperiods of the three strategies, takes an additional day of computer time per job.

4. Empirical Results

4.1 Average Returns

— **Figure 1 Goes Here** —

Panels a, b and c of Figure 1 present the empirical distributions (i.e., the “null distributions”) of the raw average returns of the randomly selected portfolios following the momentum, value and size procedures, respectively. Figures 1a-1c also show, as marked with a round bullet in each graph and reported in the caption at the bottom of each graph, the true value of the long, short, and long-minus-short sides of the momentum, value and size strategies. P-values, noted in the caption to each plot, refer to the percentage of observations in the empirical distribution that are greater than the “true value” (i.e., the value one obtains by following the true trading strategy).

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The distributions of the long and short sides of the random portfolios are similar since stocks are assigned to both portfolios randomly. Thus, the distributions of random winners and losers are nearly identical, as are those of high book-to-market and low book-to-market, as well as small and big stocks. As one would expect, the distributions of the long minus short positions appear normal and center on zero.

The true raw returns of the long side of all three investment strategies are significant positive outliers. To see this, consider for example the top left graph in Figure 1a, which shows the bell-shaped distribution of randomly-selected pseudo-winners for the momentum strategy. The true “winners” return is reported as 1.50% in the caption at the bottom of the top-left graph and is represented in this figure by the round bullet at the far right side of the top left graph in Figure 1a. Note that this 1.50% value is dramatically larger than the 1.11% mean return of the random “winners” distribution. Indeed, the P-value of 0% reported in the caption at the bottom of the top left graph in Figure 1a reveals that none of the 10,000 randomly selected pseudo-winner portfolios produced a return greater than the true winner return of 1.5%.

From Figure 1b we see that high book-to-market stocks (with a 1.42% return) also earn significantly larger profits than the corresponding random strategies. Same for small stocks in Figure 1c (true strategy return = 1.31%, which is far to the right of the null distribution). The true raw returns of the losers in Figure 1a fall far to the left of the empirical distribution, while the true raw returns of the low book-to-market stocks in Figure 1b fall close to the center of the corresponding distribution. Interestingly, the average return of the portfolio of big stocks in Figure 1c is actually larger than any random draw for the short side of the size strategy. From Figures 1a-1c we therefore conclude that momentum is driven both by high winner returns and low loser returns, whereas the value and size strategies are driven primarily by high book-to-market and small stocks, respectively.

The raw returns of the true momentum, value, and size strategies (long minus short sides) are all large outliers with p-values of essentially zero. The true return of the momentum strategy is the most distant from the mass of the empirical distribution, while the returns of the value and even more so of the size strategies are lesser outliers. Figures 1a-1c therefore starkly reveal the size, value and momentum puzzles in their raw form.

Finally, it is interesting to note that in Figure 1 (and also in the analogous figures below), the empirical distributions suggest a way of determining critical levels at which raw or risk-adjusted profits of an investment strategy can be considered truly puzzling. For instance, if we wanted to find the minimum average raw return that a strategy, such as value, must achieve before being regarded as unusual, we can examine Figure 1. The left (right) tail of the distribution ends at around -0.20% (0.20%), which implies that average monthly return that falls outside of the to $+0.20\%$ range can be ascribed to something more than pure random chance, assuming we believe that the risk adjustment (in Figure 1's case no adjustment) is appropriate.

4.2 Standard Deviations

— **Figure 2 Goes Here** —

Panels a, b and c of Figure 2 present the empirical distributions of the standard deviations of monthly returns of the long, short, and long-minus-short sides of the randomly selected portfolios for momentum, size and value strategies. The standard deviations of the true momentum, value and size portfolios are all positive outliers compared to the randomly-generated null distributions plotted in Figure 2. From Figure 2a we see that the standard deviation of losers (8.15%) is particularly large, while the standard deviation of winners (6.19%) is left of the mass of the empirical distribution. The standard deviation of the true momentum strategy (4.58%) is significantly larger than the average standard deviation for a portfolio of random winners minus losers (0.98%). From Figures 2b and 2c we see that the standard deviations of returns of the long side of both value and size strategies are larger than those of the corresponding short sides, and the resulting standard deviations of both value (3.63%) and size (3.35%) are much larger those of the random strategies.

4.3 Skewness

— **Figure 3 Goes Here** —

Figures 3a-3c present the empirical distributions of skewness of the randomly selected portfolios. From Figure 3a we see that the distribution of returns of the true winner portfolio have approximately zero skew (i.e., returns are symmetric) while the loser portfolio is positively skewed, resulting in negative skewness (-3.30) of the returns of the true winner-minus-loser momentum portfolio. While this level of skewness is unusual, about 0.34% of the random momentum portfolios have an even lower degree of asymmetry. From Figures 3b-3c we see that the portfolios of high book-to-market, low book-to-market, small, and big stocks, as well as value and size portfolios, are all positively skewed, yet neither the value nor the size portfolio skewness is a large outlier (i.e., the dots showing actual market outcomes in Figures 3b and 3c are within the empirical distributions generated under the null hypothesis of no strategy effect).

4.4 Kurtosis

— **Figure 4 Goes Here** —

Figures 4a-4c present the empirical distributions of kurtosis of the randomly selected portfolios. Figure 4a reveals that kurtosis of the winners is noticeably lower (left side of the distribution) while that of losers is higher, than the kurtosis of any random winner or loser portfolio; the net long-short portfolio has higher kurtosis. Figure 4b shows that kurtosis of the low book-to-market stocks falls close to the center of the empirical distribution, whereas for the portfolios of high book-to-market, small, and big stocks (in Figure 4c), it is significantly large. Interestingly, in all cases the kurtosis of the long-minus-short strategies is larger than the mean kurtosis of the random portfolios but is in no case a dramatic outlier. The empirical distribution of the kurtosis of the random long minus short position is also curious in that it is clearly skewed to the right rather than being symmetric.

4.5 Ratio of Average Returns to Standard Deviations

— **Figure 5 Goes Here** —

High returns and large standard deviations suggest that the high profits of momentum, value, and size may be providing compensation for the volatility of returns to these strategies. This motivates us to examine the ratio of average returns to standard deviations – i.e., Sharpe ratios – which are presented in Figures 5a-5c. While winners, losers, and low book-to-market stocks are significant outliers, the true values of the Sharpe ratios of high book-to-market, small, and big stocks fall in the mass of the empirical distributions. Interestingly, the true values of the Sharpe ratios of the three strategies, while somewhat unusual, do not appear extremely uncommon: while the Sharpe ratio of the true momentum strategy is a marginal outlier, 0.03% of the random high minus low portfolios have a higher Sharpe ratio than the true value portfolio, with the comparable number for the size strategy equaling 2.37%. The true values thus do fall in the tail of the empirical distribution, yet none of them are extreme outliers. Figures 5a-5c thus suggest that a part of the profits from the three investment strategies (i.e., momentum, value and size) could be related to a premium for high volatility of returns relative to mean returns, as captured by the Sharpe ratios.

4.6 Ratio of Average Returns to Skewness

— **Figure 6 Goes Here** —

Figures 6a-6c plot ratios of mean returns to skewness. From these figures we see that, out of the nine portfolios under investigation, only winners (in Figure 6a) whose returns are almost perfectly symmetric, and losers, have a ratio of average-returns-to-skewness that falls outside of their empirical distributions. The ratio of returns to skewness of the loser portfolio in Figure 6a just misses the left tail of the distribution. For the long and short sides of value and size portfolios (Figures 6b-6c), and more importantly for all three long minus short portfolios, the level of average returns to skewness is far from unusual as many

of the random portfolios have either a much higher, or much lower, ratio. This evidence provides support for the conclusions of Harvey and Siddique (2000) whose skewness factor explains a portion of strategy profits, and motivates further research into the compensation for skewness of portfolio returns (which is the subject of Section 5 below). If investors demand a high premium for holding portfolios with high skewness, at least some fraction of the profits from momentum, value and size investing can be explained.

4.7 Ratio of Average Returns to Kurtosis

— **Figure 7 Goes Here** —

In Figures 7a-7c we examine the distribution of the ratios of average monthly returns to kurtosis of the three strategies. The true winner portfolio exhibits an unusually high ratio (Figure 7a). Conversely, portfolios of losers, high book-to-market stocks (Figure 7b) and small and large stocks (Figure 7c) have a ratio of average returns to kurtosis that falls in the left tail of the distribution. From Figure 7a we see that, for the true WL momentum portfolio, the return-to-kurtosis ratio 0.02 falls in the tail of the empirical distribution, with 0.19% of random draws producing a higher ratio. Likewise, for both value (Figure 7b) and size (Figure 7c) portfolios, the ratios (0.03 and 0.01 respectively) are not unusual. Similar to the cases with the Sharpe ratio and the returns-to-skewness ratio, Figure 7's evidence suggests that the high profits of the three investment strategies may be related to the excessive kurtosis of the momentum, value and size portfolios. In particular, an asset pricing model that incorporates compensation for holding portfolios with high kurtosis has the potential to explain at least a fraction of the high profits of the three investment strategies.

4.8 Market Model Risk Adjustment

— **Figure 8 Goes Here** —

It has been documented in prior literature that the unconditional CAPM cannot explain any of the three puzzles we examine (i.e., momentum, value and size). In Figures 8a-8c, we

present the empirical distributions of the alphas from the market model regression:

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it},$$

where R_{it} and R_{Mt} are month t excess returns of portfolio i and the value-weighted market index from CRSP, respectively, using the risk-free rate from Ken French's data library.

From Figures 8a-8c we see that the long sides of the three strategies have unusually high alphas, as does the short side of the size portfolio. Losers and low book-to-market stocks have low alphas. Consistent with findings in prior literature, we find that the market risk-adjusted returns of the momentum strategy (momentum alpha = 0.95%) are higher than raw returns, aggravating the puzzle. In other words, adjusting for risk using the CAPM makes the momentum puzzle even more puzzling. The CAPM does explain a fraction of returns to the value and size strategies, whose alphas (0.35% and 0.10% respectively) are lower than the raw returns. Alpha of the size strategy does fall in the tail of the distribution, with 0.24% of random portfolios producing higher risk-adjusted profits than the size strategy, calling into question the significance of its performance.

4.9 Multi-Factor Model Alphas When Own-Factor Is Omitted

— **Figure 9 Goes Here** —

In Figures 9a-9c we present the empirical distributions of alphas from the regressions of the form

$$R_{it} = \alpha_i + \sum_F \beta_{Fi} R_{Ft} + \varepsilon_{it},$$

where β_{Fi} and R_{Ft} are loadings on, and returns of, factor F , respectively. For the momentum strategy, factors include excess return on market, HML, and SMB factors, obtained from Ken French's data library, but do not include a momentum factor. Analogously, for the value strategy we include market and size factors but exclude the HML value factor, while for

the size strategy we omit the SMB factor but include market and value factors. That is, for each puzzle, we exclude the factor that is constructed to explain such puzzle. The results of Figures 9a-9c show that the returns from the long minus short position of each investment strategy remain significantly large and are outliers. Value and size strategy alphas (0.34% and 0.08%) are similar to those obtained using a simple market model (0.35% and 0.10%). Consistent with the prior literature, the 3-factor risk adjustment augments the momentum profits, again increasing the puzzle, mainly by decreasing the loser excess returns; 3-factor momentum alpha is 1.11%, well above its market model counterpart of 0.95%.

4.10 Multi-Factor Model Alphas When Own-Factor Is Included

— **Figure 10 Goes Here** —

Figure 10 presents the empirical distribution of alphas from the regressions of the form

$$R_{it} = \alpha_i + \sum_F \beta_{Fi} R_{Fi} + \varepsilon_{it},$$

where for value and size strategies F are the Fama-French factors, and for momentum strategy F includes the momentum factor UMD, from Ken French's data library, in addition to the standard 3 factors of Fama-French. In other words, all factors are now included in every case. Inclusion of the UMD factor decreases momentum alpha to a statistically insignificant 0.06%, which falls within the 95% of the mass of the empirical distribution of alphas, and can therefore not be considered unusually high. Not surprisingly, since the factor models are in a sense designed to account for the puzzles, the 3-factor model-adjusted profits of size and value are close to zero and can therefore not be considered unusual after this particular risk-adjustment is undertaken.

4.11 Betas and Subperiod Robustness

For the ‘market model’ in Section 4.8, the ‘multifactor model with own-factor omitted’ in Section 4.9, and the ‘multifactor model with own-factor included’ in Section 4.10, we investigate the behavior of the factor betas, in addition to the investigating the alphas as already discussed in Sections 4.8 through 4.10 above and plotted in Figures 8 through 10.

Since the UMD, HML and SMB factors employed in Section 4.10 are specifically constructed to capture the momentum, value and size effects respectively, we would expect each of these three factors to capture the risk they were constructed to capture. For example, in the case of the momentum strategy we would expect the beta on the UMD factor in the ‘multifactor model with own-factor included’ regression in Section 4.10 to be close to unity and the other betas to have values around zero, and indeed this is what we find; i.e., the UMD factor captures momentum risk as intended. Similarly, for the value strategy would expect the HML loading to be close to unity with the other betas around zero, and for the size strategy the SMB loading to be close to unity with the other betas around zero, and in both cases this is what we found in Section 4.10’s regressions (detailed results are available on request).

Conversely, and more surprisingly, in Section 4.9’s regressions where the own-factor is omitted, and in Section 4.8’s simple market model, the betas associated with momentum, value and size investment strategies all lie far outside their expected ranges. For example, in Section 4.8’s market model regressions, we find that the true momentum portfolio produces a large negative beta from the market model estimation (much more negative than any of our 10,000 random simulations), while the true value and size portfolios produce much larger positive market-model betas than any of our random simulations. This reveals that any of the risk corrections commonly employed in the literature do not adequately explain excess profits from the momentum, size and value puzzles. This finding, along with our previous

finding that returns to momentum, size and value trading strategies can be accounted for in part by volatility, kurtosis and especially skewness of returns to such strategies, suggests that additional investigation of volatility, kurtosis and especially skewness is in order. Such an investigation is the subject of Section 5 below.

Before turning to a deeper investigation of skewness and kurtosis, we first check the robustness of our findings from Sections 4.1 to 4.10 above. To check robustness, we re-performed our entire analysis on a variety of 20-year and 40-year subperiod partitions of our full data sample. We found essentially the same results as those reported above for the full 80-year sample, thereby lending confidence to the robustness of our results. Perhaps most importantly, consistent with the above-reported findings for the full 80-year period, for each subperiod and each investment strategy (i.e., momentum, value and size), we found that the ratios of average returns to standard deviation, average returns to skewness, and average returns to kurtosis, all tend to generally fall somewhere in the mass of the empirical distributions. For example, although profits from momentum trading are unusually high compared to random portfolios, and although the momentum portfolio's skewness is also unusually high relative to random portfolios, the ratio of return to skewness in the momentum portfolio looks somewhat similar to the return/skewness ratio from randomly generated portfolios we documented in Section 4.6 above. This strengthens the possibility that profits from momentum, value and size investing relate in some way to the levels of volatility and particularly skewness and kurtosis of their returns and motivates further research into the compensation that investors demand for holding such portfolios. We now turn to such an investigation.

5. Roles of Skewness and Kurtosis in Explaining Trading Strategy Profits

The results presented in the prior sections of this paper suggest that the profitability of the three investment strategies under consideration may be attributable at least in part to risk associated with skewness and kurtosis. In this section we construct several skewness and kurtosis factors and assess their ability to aid the explanation of profits from momentum, value, and size investing.

5.1 Constructing Skewness and Kurtosis Factors

The spirit of our approach follows Harvey and Siddique (2000), who compute past coskewness of stocks and show, among other things, that a value-weighted portfolio of the 30% of NYSE/AMEX stocks with the lowest coskewness helps to explain part of the profits from long-short investing. We recreate the coskewness factor of Harvey and Siddique (2000) and then also follow a similar methodology to create several kurtosis factors and alternative skewness factors. More precisely, we compute standardized unconditional coskewness for asset i in month τ as: $E(\varepsilon_{it}\varepsilon_{Mt}^2) / \sqrt{E(\varepsilon_{it}^2)E(\varepsilon_{Mt}^2)}$, where ε_{it} are residuals from regressing the excess return of asset i on the market return using monthly return data from month $\tau - 60$ to $\tau - 1$. The coskewness factor for month τ , noted $SK_{HS,\tau}$, is the value-weighted average return of the 30% of stocks with the lowest coskewness.

Kim and White (2004) discuss robust measures of skewness and kurtosis, which we use to construct several factors and investigate their ability to explain profits from the investment strategies in question. In particular, for each stock i and month τ , we use the data from

months $t \in \{\tau - 60, \tau - 1\}$ to calculate the following four measures of skewness:

$$\begin{aligned}
SK_{1,i,\tau} &= E \left(\frac{R_{it} - \mu_i}{\sigma_i} \right)^3, \text{ where } R_{it} \text{ is the return of stock } i \text{ in month } t, \mu_i \text{ is the mean} \\
&\quad \text{60-month return and } \sigma_i \text{ is the standard deviation of returns over this period;} \\
SK_{2,i,\tau} &= \frac{Q_{i,3} + Q_{i,1} - 2Q_{i,2}}{Q_{i,3} - Q_{i,1}}, \text{ where } Q_{i,j} \text{ is the } j\text{th quintile of } R_{it}; \\
SK_{3,i,\tau} &= \frac{\mu_i - Q_{i,2}}{E |R_{it} - Q_{i,2}|}; \\
SK_{4,i,\tau} &= \frac{\mu_i - Q_{i,2}}{\sigma_i}.
\end{aligned}$$

We also calculate the following four measures of kurtosis for each stock i and month τ , again using the data from months $t \in \{\tau - 60, \tau - 1\}$:

$$\begin{aligned}
KR_{1,i,\tau} &= E \left(\frac{R_{it} - \mu_i}{\sigma_i} \right)^4 - 3; \\
KR_{2,i,\tau} &= \frac{(O_{i,7} - O_{i,5}) + (O_{i,3} - O_{i,1})}{O_{i,6} - O_{i,2}} - 1.23, \text{ where } O_{i,j} \text{ is the } j\text{th octile of } R_{it}; \\
KR_{3,i,\tau} &= \frac{U_{i,5} - L_{i,5}}{U_{i,50} - L_{i,50}} - 2.59, \text{ where } U_{i,j} (L_{i,j}) \text{ is the average of the upper (lower)} \\
&\quad j \text{ percentiles of } R_{it}; \\
KR_{4,i,\tau} &= \frac{P_{i,97.5} - P_{i,2.5}}{P_{i,75} - P_{i,25}} - 2.91, \text{ where } P_{i,j} \text{ is the value of the } j\text{th percentile of } R_{it}.
\end{aligned}$$

The four skewness factors, SK_1 through SK_4 , are defined similarly to Harvey and Siddique (2000): as a value-weighted return on the portfolio of the 30% of NYSE/AMEX stocks with the lowest skewness measures. The kurtosis factors we explore are defined as either a value-weighted return on the portfolio of 30% of NYSE/AMEX stocks with the highest kurtosis (noted KR_1^H through KR_4^H) or with the lowest kurtosis (noted KR_1^L through KR_4^L). We now explore the ability of these factors to explain profits from the momentum, value and size investment strategies.

5.2 Empirical Results

Table 2 presents the results from regressions of returns from momentum, value and size investing on our various skewness and kurtosis factors added to a simple market model; i.e.,

$$R_{it} = \alpha_i + \beta_{Mi}R_{Mt} + \sum_F \beta_{Fi}R_{Fi} + \varepsilon_{it},$$

where R_{it} is the return on the momentum, value or size portfolio, as the case may be, R_{Mt} is the market return, and R_{Fi} are the various skewness and kurtosis factors described above.

— **Table 2 Goes Here** —

Panel A of Table 2 reports results from regressing the returns from momentum investing on the market return and various skewness and kurtosis factors. Here we see that, although alpha retains statistically significant t -statistics [reported in square brackets], no matter what measures of skewness we add as factors to a simple market model of momentum strategy returns, the skewness factors are themselves in general strongly significant. This reveals that the skewness factors play some important role in accounting for momentum strategy profits.¹¹ Furthermore, the adjusted R^2 increases from 0.116 for the basic market model up to as much as 0.242 when we add the skewness factor SK_4 . Results from adding kurtosis factors deliver a similar message: while they do not eliminate momentum profits (i.e., eliminate the significance of alpha), the loadings are strongly significant and inclusion of kurtosis factors can dramatically increase the adjusted R^2 compared to a simple market model regression (e.g. R^2 rises from 0.116 in the basic market model up to 0.275 when including kurtosis factor KR_4^L).

Panel B of Table 2 reports results from regressing the returns from value investing on the market return and various skewness and kurtosis factors. Note that not only are the loadings

¹¹The SK_{HS} factor of Harvey and Siddique (2000) that we reconstruct proves useful in their sample covering the years 1963-1993, but does not help explain momentum profits in our broader 1926-2006 sample.

on our skewness and kurtosis factors generally highly significant, but that adding almost any one of the skewness and kurtosis factors eliminates the significance of value strategy profits; i.e., alpha becomes statistically insignificant in many cases when a skewness or kurtosis factor is included. For example, adding skewness factor SK_2 to the market model renders the value strategy alpha statistically insignificant and increases the R^2 from 0.053 to 0.221. Similarly, including kurtosis factor KR_2^H to the market model decreases alpha from 0.34% to an insignificant -0.05% and increases R^2 to 0.239. All skewness factors except SK_{HS} are significantly positive, and all kurtosis factors except KR_4^H are significantly positive.

Panel C of Table 2 reports results from regressing the returns from size investing on the market return and various skewness and kurtosis factors. The conclusions we can draw from this panel are largely similar to those drawn for momentum. While all but three regressions fail to eliminate size profits, inclusion of skewness and kurtosis factors increases adjusted R^2 when compared with simple market model regressions. Loadings on all skewness and KR^L (low kurtosis) factors are significantly negative, while loadings on KR^H (high kurtosis) factors are generally positive.

— **Table 3 Goes Here** —

To further investigate the ability of skewness and kurtosis factors to explain profits from investment strategies, we construct two more factors. We define a LMH (low minus high) skewness factor as the value-weighted return on a portfolio of NYSE/AMEX stocks with measures of SK_3 in the lowest three deciles minus a value-weighted return on a portfolio with measures of SK_3 in the highest three deciles. We similarly define a kurtosis LMH (low minus high) factor as the value-weighted return on a portfolio of NYSE/AMEX stocks with measures of KR_3 in the lowest three deciles minus a value-weighted return on a portfolio with measures of KR_3 in the highest three deciles. We present the results of adding these two factors to a simple market model in Table 3. While the factors do not eliminate the

profits from the three investment strategies we consider, the loadings are highly significant and adjusted R^2 increase dramatically compared to the market model.

We can interpret the results presented in Tables 2 and 3 as evidence that constructed skewness and kurtosis factors can help explain the profits, and variation in profits, from the three investment strategies under consideration: value, size and momentum. The loadings on the skewness and kurtosis factors we construct are generally highly significant and inclusion of these factors improves the proportion of variability of investment profits (as measured by R^2) explained as compared to a simple market model. In the case of the value strategy, inclusion of even the simple factors that we consider eliminates the significance of the strategy trading profits.

6. Conclusion

This paper employs a simulation-based methodology to evaluate the significance of profits from various popular long-short investment strategies and investigates the sources of risk in such strategies. The methodology calls for generating distributions of ‘random’ raw and risk-adjusted returns obtained by mimicking the portfolio formation and holding of a particular strategy, but selecting stocks into portfolios randomly rather than using the specified strategic criteria. The resulting empirical distributions can then be used under the null hypothesis of no strategy effect to evaluate profits, risk measures and risk-adjusted returns of the investment strategy and to draw inferences about their significance and implications regarding sources of the puzzles.

We apply the methodology to momentum, value and size puzzles. We generate empirical distributions of the mean, standard deviation, skewness and kurtosis of monthly returns of the random portfolios and compare them to the appropriate moments of the true returns of the three investment strategies. We also obtain empirical distributions of the ratio of average

returns to standard deviation, skewness and kurtosis, as well as market and multifactor alphas and loadings.

Our findings reveal that significant insights can be gained from a Monte Carlo approach to evaluating long-short trading strategies. For example, our results reveal that if skewness and kurtosis are priced, the puzzles behind the large profits of momentum, value and size investing can be explained, or at least alleviated substantially.

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Tables and Figures

Table 1

	MOMENTUM			VALUE			SIZE		
	WIN	LOS	WL	VALUE	GROWTH	HML	SMALL	BIG	SMB
1927-2006	1.50	0.73	0.77	1.42	0.96	0.46	1.31	1.07	0.24
1927-1966	1.49	0.80	0.69	1.47	1.00	0.46	1.36	1.06	0.30
1967-2006	1.50	0.67	0.84	1.37	0.91	0.46	1.25	1.08	0.17
1927-1946	1.45	0.97	0.49	1.63	1.03	0.61	1.62	0.97	0.65
1947-1966	1.54	0.64	0.90	1.30	0.98	0.32	1.11	1.15	-0.05
1967-1986	1.50	0.67	0.83	1.48	0.92	0.56	1.36	1.04	0.32
1987-2006	1.51	0.66	0.84	1.26	0.91	0.35	1.15	1.12	0.03

Table 1. This table reports the average raw returns, in percent per month, of the momentum, value, and size strategies in different subperiods. Details of portfolio formation are outlined in Section 2 of this paper. The sample period is 1931 to 2006.

Table 2

PANEL A – MOMENTUM STRATEGY						
Regression	Alpha	Market	Skew	Kurtosis ^L	Kurtosis ^H	R ²
Mean	0.686 [4.40]					
Market Model	0.884 [5.99]	-0.298 [10.96]				0.116
SK ₁	1.224 [7.97]	0.629 [4.34]	-0.975 [6.50]			0.154
SK ₂	1.272 [8.50]	0.859 [6.13]	-1.148 [8.40]			0.179
SK ₃	1.361 [9.43]	1.184 [8.94]	-1.503 [11.40]			0.226
SK ₄	1.363 [9.60]	1.267 [9.79]	-1.572 [12.33]			0.242
SK _{HS}	0.905 [6.00]	-0.216 [2.41]	-0.068 [0.73]			0.102
KR ₁	1.125 [7.31]	0.395 [2.74]		-0.695 [4.89]		0.138
KR ₁	0.977 [6.61]	0.147 [1.36]			-0.419 [4.25]	0.132
KR ₂	0.932 [5.80]	-0.163 [0.90]		-0.136 [0.75]		0.115
KR ₂	1.078 [7.20]	0.341 [2.81]			-0.643 [5.41]	0.142
KR ₃	0.998 [6.37]	0.044 [0.27]		-0.349 [2.14]		0.119
KR ₃	1.060 [7.35]	0.464 [4.80]			-0.726 [8.20]	0.176
KR ₄	1.337 [9.73]	1.174 [10.98]		-1.533 [14.15]		0.275
KR ₄	0.714 [4.86]	-0.897 [9.24]			0.555 [6.41]	0.153

Table 2. This table reports coefficients and t-statistics from regressions of returns from momentum (in Panel A), value (in Panel B), and size (in Panel C) investing on excess market returns and the noted skewness and kurtosis factors. The sample period is 1931 to 2006.

TABLE 2: PANEL B - VALUE STRATEGY

Regression	Alpha	Market	Skew	Kurtosis ^L	Kurtosis ^H	R ²
Mean	0.445 [3.71]					
Market Model	0.342 [2.90]	0.156 [7.19]				0.053
SK ₁	0.241 [1.93]	-0.118 [1.00]	0.288 [2.36]			0.057
SK ₂	-0.145 [1.29]	-1.296 [12.31]	1.440 [14.03]			0.221
SK ₃	-0.022 [0.19]	-0.976 [9.17]	1.147 [10.83]			0.160
SK ₄	0.025 [0.22]	-0.875 [8.26]	1.036 [9.92]			0.144
SK _{HS}	0.355 [2.92]	0.164 [2.28]	-0.018 [0.24]			0.046
KR ₁	0.123 [1.01]	-0.470 [4.11]		0.629 [5.57]		0.083
KR ₁	0.176 [1.56]	-0.635 [7.67]			0.745 [9.87]	0.144
KR ₂	0.245 [1.91]	-0.115 [0.80]		0.273 [1.90]		0.055
KR ₂	-0.049 [0.45]	-1.127 [12.80]			1.291 [14.94]	0.239
KR ₃	0.252 [2.02]	-0.110 [0.85]		0.272 [2.08]		0.056
KR ₃	0.141 [1.27]	-0.714 [9.65]			0.829 [12.22]	0.186
KR ₄	0.075 [0.65]	-0.710 [7.93]		0.902 [9.94]		0.145
KR ₄	0.361 [3.01]	0.223 [2.81]			-0.062 [0.87]	0.053

Table 2. Continued.

TABLE 2: PANEL C - SIZE STRATEGY

Regression	Alpha	Market	Skew	Kurtosis ^L	Kurtosis ^H	R ²
Mean	0.300 [2.68]					
Market Model	0.161 [1.51]	0.209 [10.70]				0.111
SK ₁	0.693 [6.89]	1.658 [17.47]	-1.524 [15.52]			0.297
SK ₂	0.509 [4.83]	1.248 [12.65]	-1.031 [10.71]			0.210
SK ₃	0.576 [5.74]	1.499 [16.27]	-1.308 [14.26]			0.273
SK ₄	0.522 [5.15]	1.389 [15.04]	-1.185 [13.02]			0.250
SK _{HS}	0.324 [3.05]	0.628 [9.94]	-0.443 [6.81]			0.161
KR ₁	0.510 [4.80]	1.212 [12.20]		-1.006 [10.27]		0.203
KR ₁	0.070 [0.66]	-0.225 [2.91]			0.409 [5.81]	0.142
KR ₂	0.701 [6.58]	1.729 [14.41]		-1.530 [12.81]		0.246
KR ₂	0.213 [1.95]	0.383 [4.33]			-0.174 [2.01]	0.114
KR ₃	0.573 [5.41]	1.438 [13.12]		-1.254 [11.37]		0.221
KR ₃	0.019 [0.19]	-0.402 [5.84]			0.583 [9.23]	0.186
KR ₄	0.403 [3.88]	0.996 [12.34]		-0.819 [10.02]		0.198
KR ₄	0.014 [0.13]	-0.308 [4.45]			0.479 [7.77]	0.165

Table 2. Continued.

Table 3

Model	Alpha	Market	Skew ^{LMH}	Kurtosis ^{LMH}	R ²
MOMENTUM					
Mean	0.686 [4.40]				
Market Model	0.884 [5.99]	-0.298 [10.96]			0.116
Skew & Kurtosis Factors Added	0.835 [5.96]	-0.341 [12.43]	-0.644 [8.52]	0.425 [6.29]	0.204
VALUE					
Mean	0.445 [3.71]				
Market Model	0.342 [2.90]	0.156 [7.19]			0.053
Skew & Kurtosis Factors Added	0.390 [3.51]	0.167 [7.66]	0.393 [6.54]	-0.478 [8.92]	0.155
SIZE					
Mean	0.300 [2.68]				
Market Model	0.161 [1.51]	0.209 [10.70]			0.111
Skew & Kurtosis Factors Added	0.194 [2.14]	0.104 [5.84]	-0.618 [12.61]	-0.526 [12.02]	0.350

Table 3. This table reports coefficients and t-statistics from regressions of returns from momentum, value, and size investing on excess market and skewness and kurtosis factors. Skewness factor (LMH, low minus high) is defined as the value-weighted return on a portfolio of NYSE/AMEX stocks with measures of SK3 in the lowest three deciles minus a portfolio with measures of SK3 in the highest three deciles. Kurtosis factor is defined similarly using the KR3 measure. The sample period is 1931 to 2006.

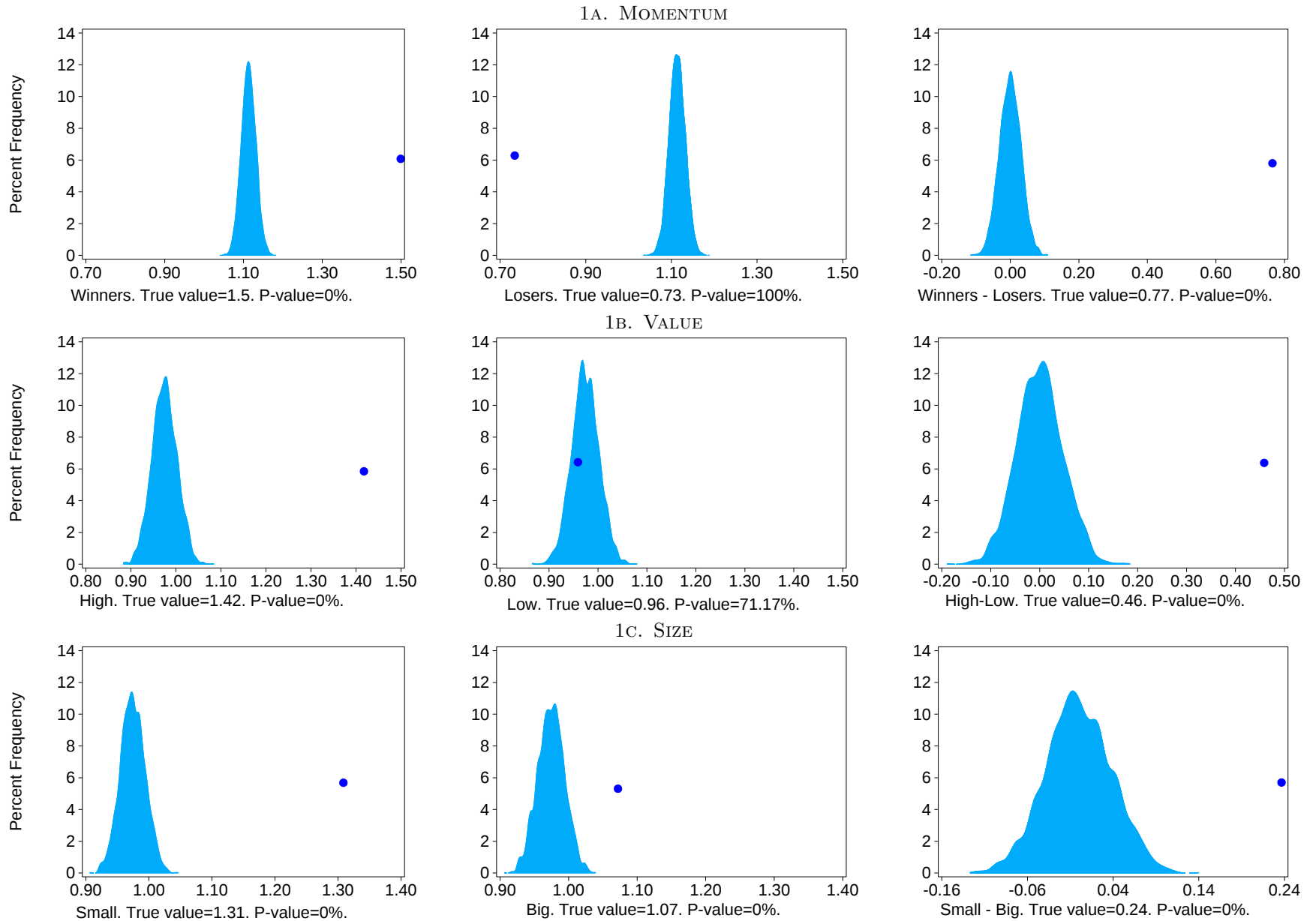


Figure 1: Average Returns. This figure plots the empirical distributions of average returns, in percent per month, of the randomly selected portfolios. The portfolios are formed in precisely the same manner as the true momentum, value, and size portfolios, except that stocks are chosen randomly. The distributions of average monthly returns for the 1927-2007 period from 10,000 randomly specified portfolios are then plotted for the long, short, and long minus short positions. The true average monthly returns of each portfolio are noted in the horizontal axis caption, and are indicated by a dot in each figure. P-value refers to the percent of the random observations that are greater than the true average monthly return.

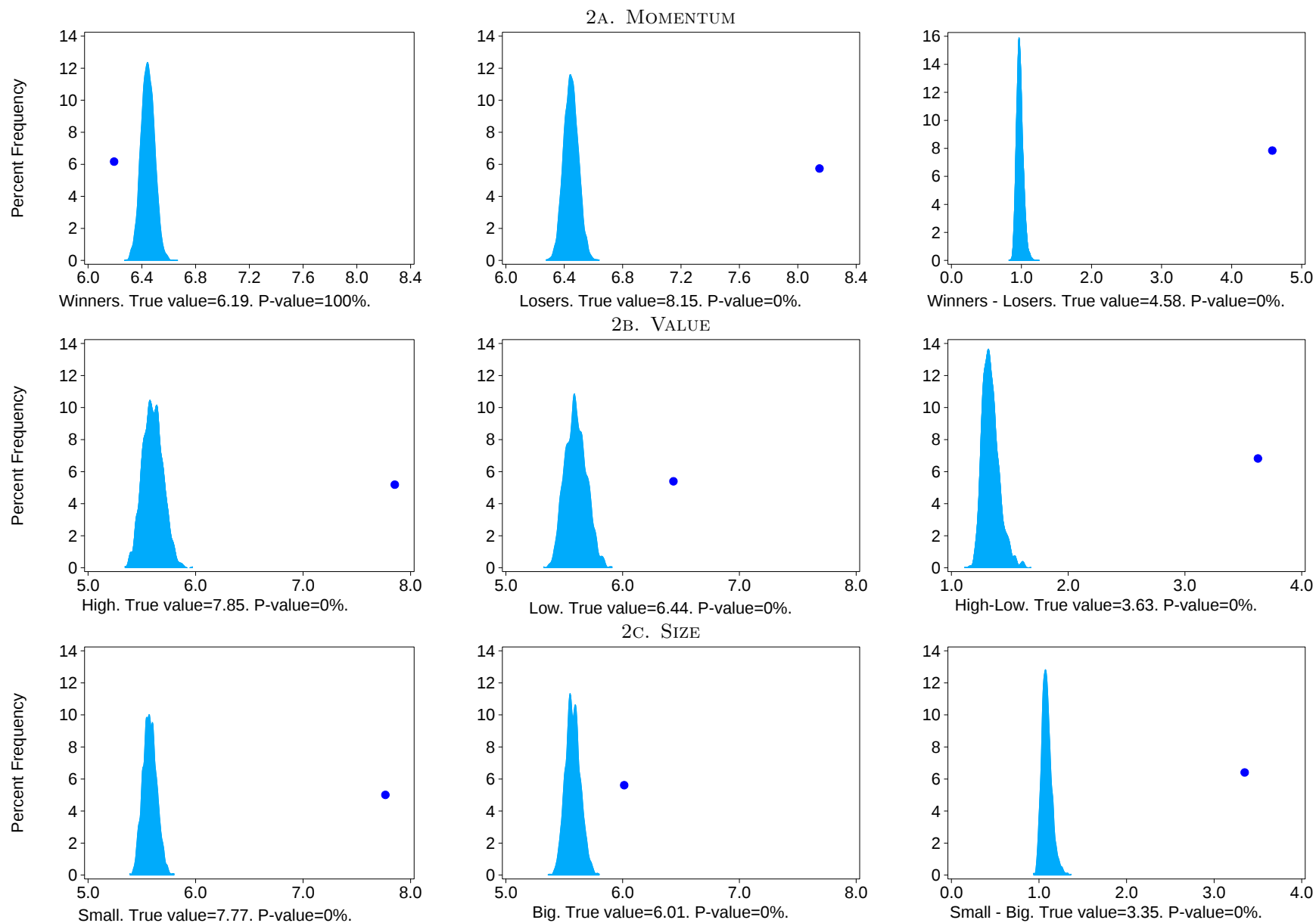


Figure 2: Standard Deviation. This figure plots the empirical distributions of standard deviations of returns of the randomly selected portfolios. The true standard deviations of each strategy portfolio are noted in the horizontal axis caption, and are indicated by a dot in each figure. P-value refers to the percent of the random observations that are greater than the true average monthly standard deviation.

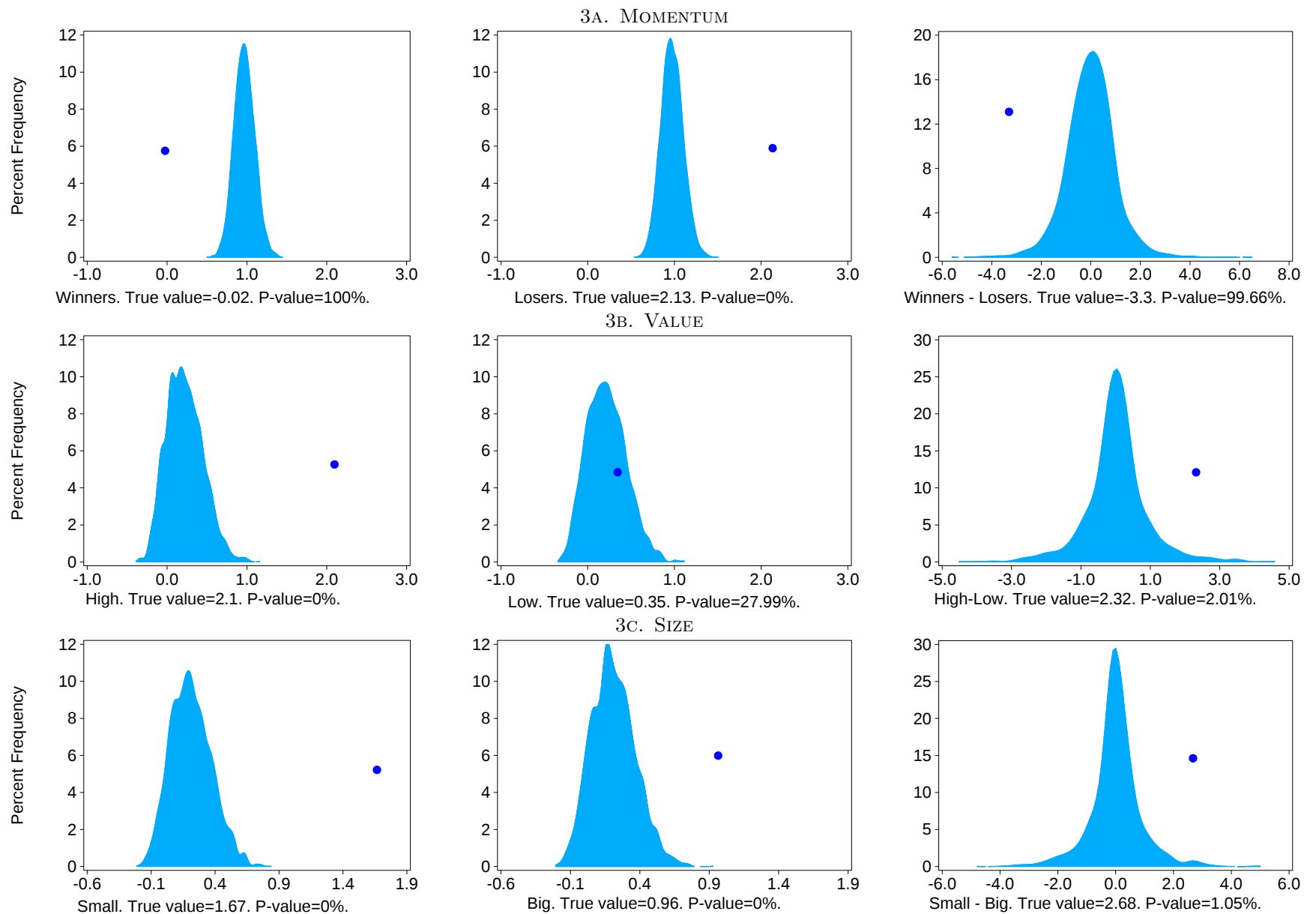


Figure 3: Skewness. This figure plots the empirical distributions of skewness of returns of the randomly selected portfolios. The true skewness of each strategy portfolio is noted in the horizontal axis caption, and is indicated by a dot in each figure. P-value refers to the percent of the random observations that are greater than the true skewness.

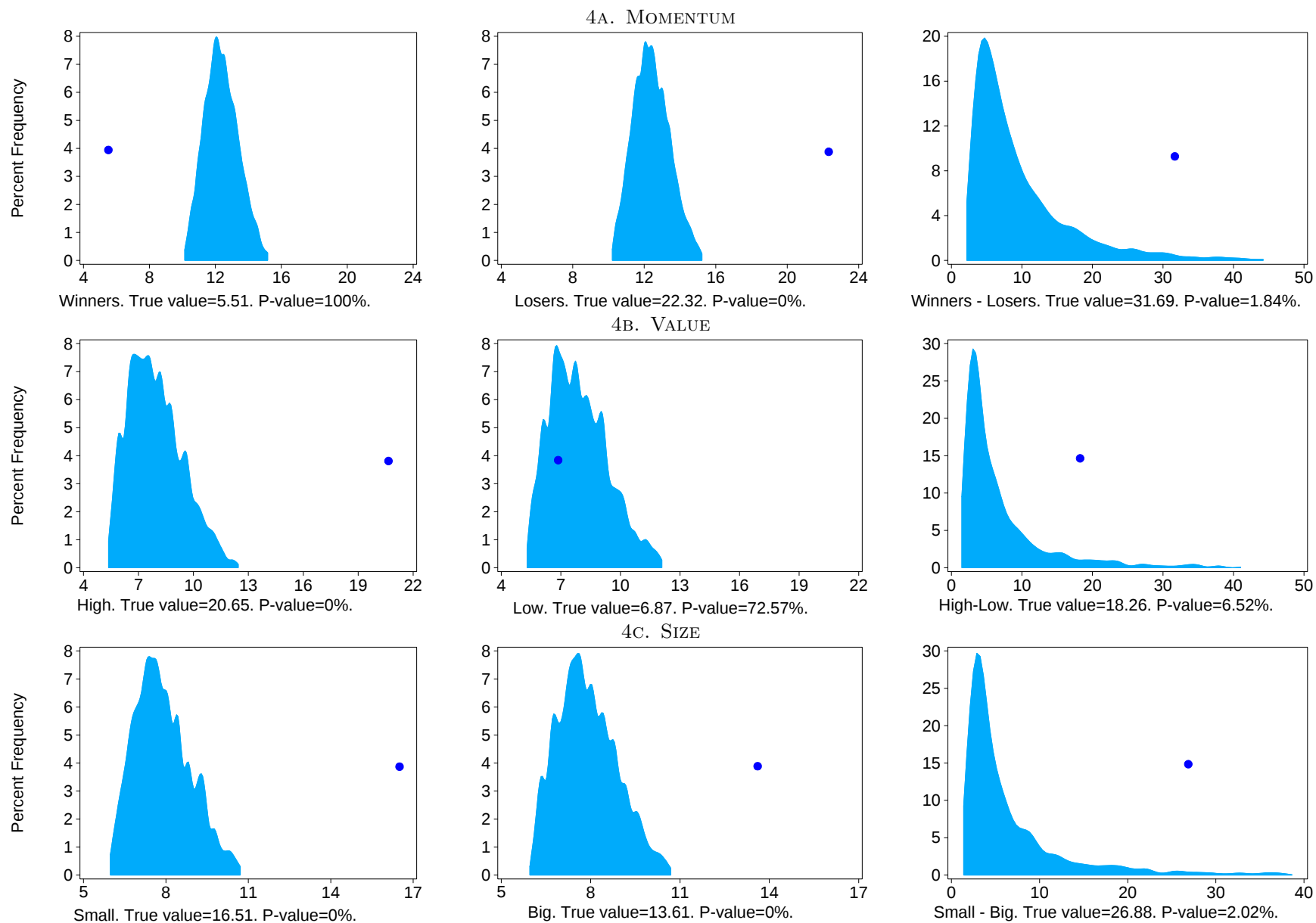


Figure 4: Kurtosis. This figure plots the empirical distributions of kurtosis of returns of the randomly selected portfolios. The true kurtosis of each strategy portfolio is noted in the horizontal axis caption, and is indicated by a dot in each figure. P-value refers to the percent of the random observations that are greater than the true kurtosis.

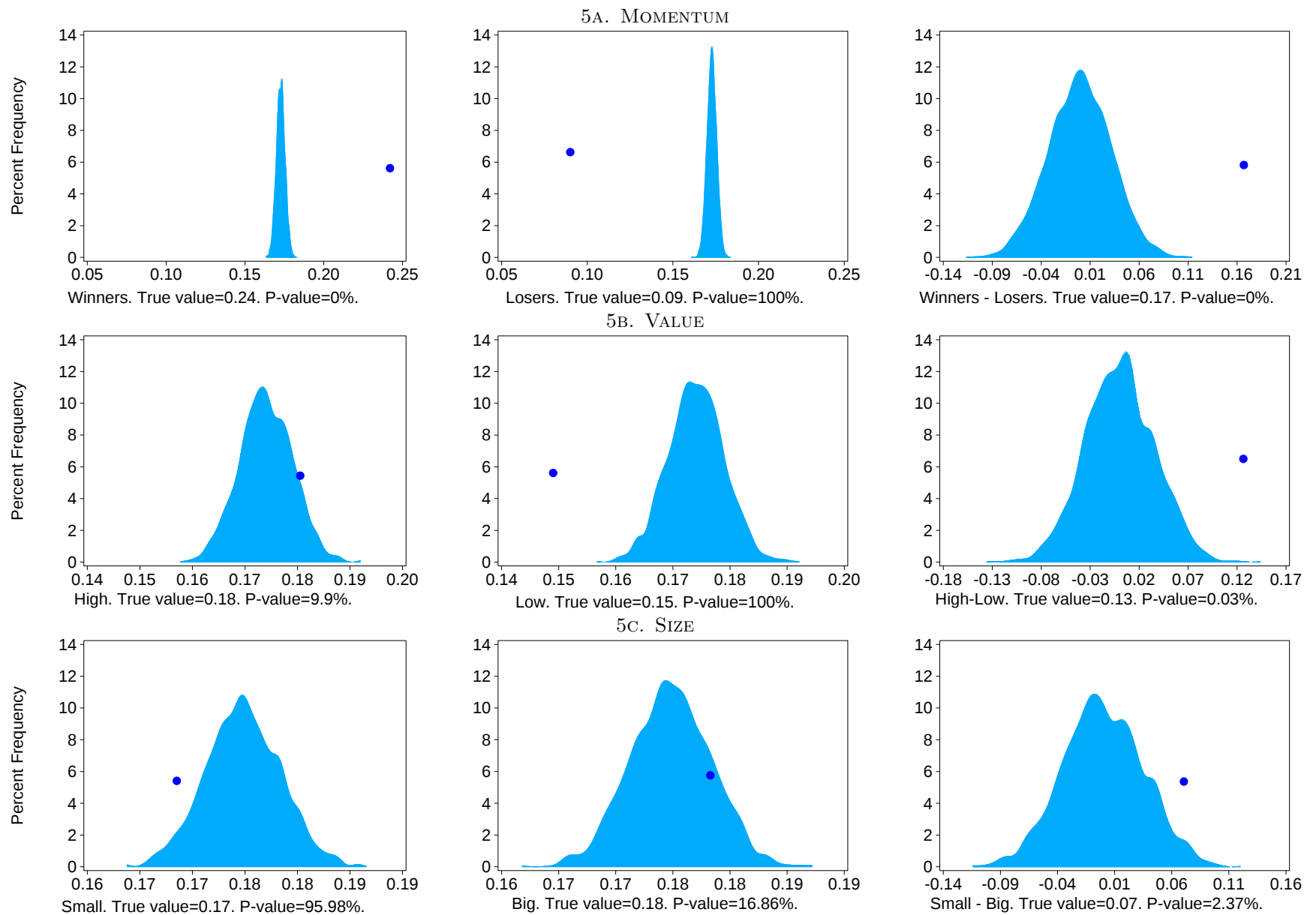


Figure 5: Sharpe Ratio. This figure plots the empirical distributions of the ratios of average monthly returns to standard deviation (Sharpe ratio) of the randomly selected portfolios. The true Sharpe ratio of each strategy portfolio is noted in the horizontal axis caption, and is indicated by a dot in each figure. P-value refers to the percent of the random observations that are greater than the true Sharpe ratio.

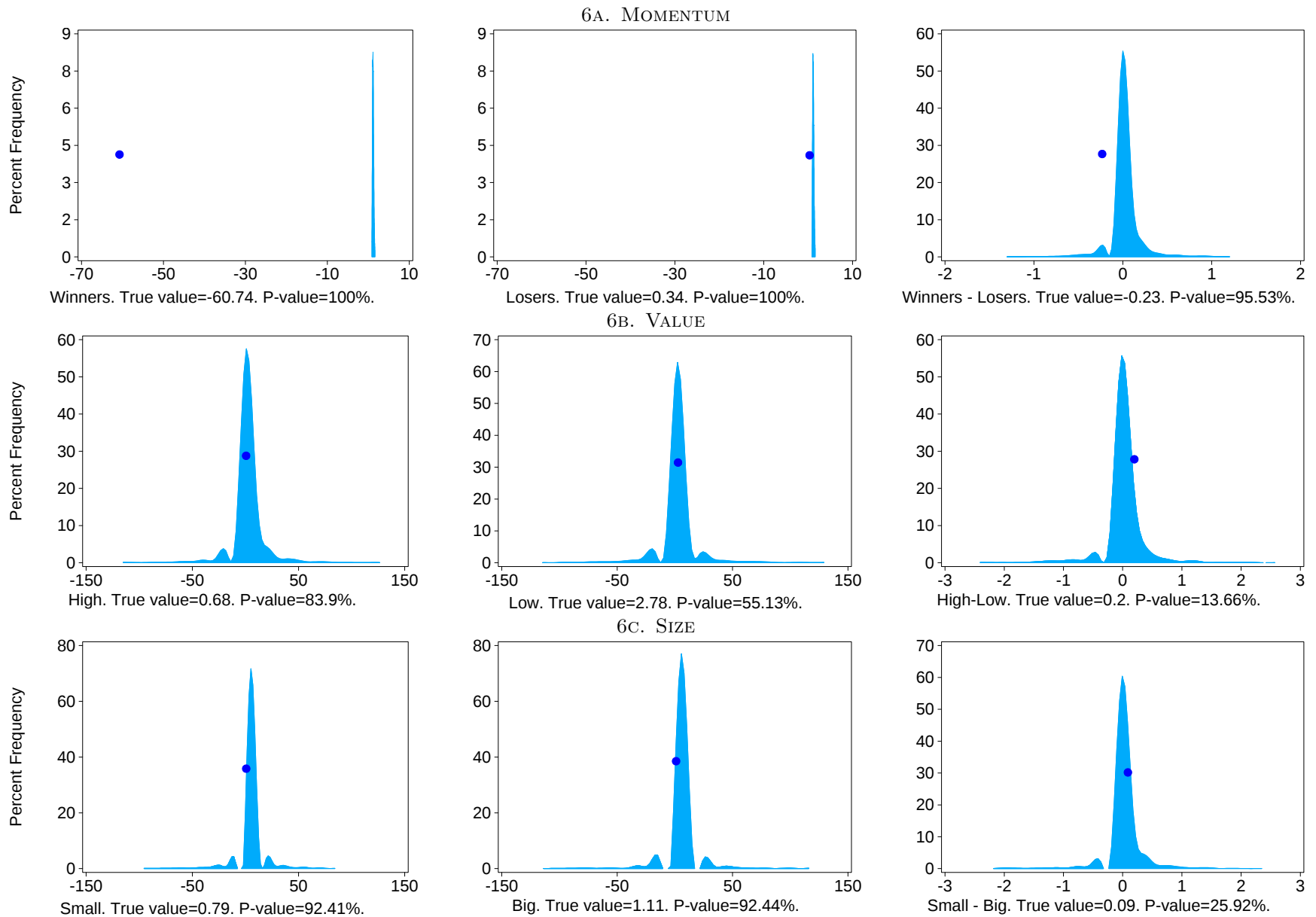


Figure 6: Ratio of Average Return to Skewness. This figure plots the empirical distributions of the ratios of average monthly returns, in percent per month, to skewness of the randomly selected portfolios. The true ratio of each strategy portfolio is noted in the horizontal axis caption, and is indicated by a dot in each figure. P-value refers to the percent of the random observations that are greater than the true ratio.

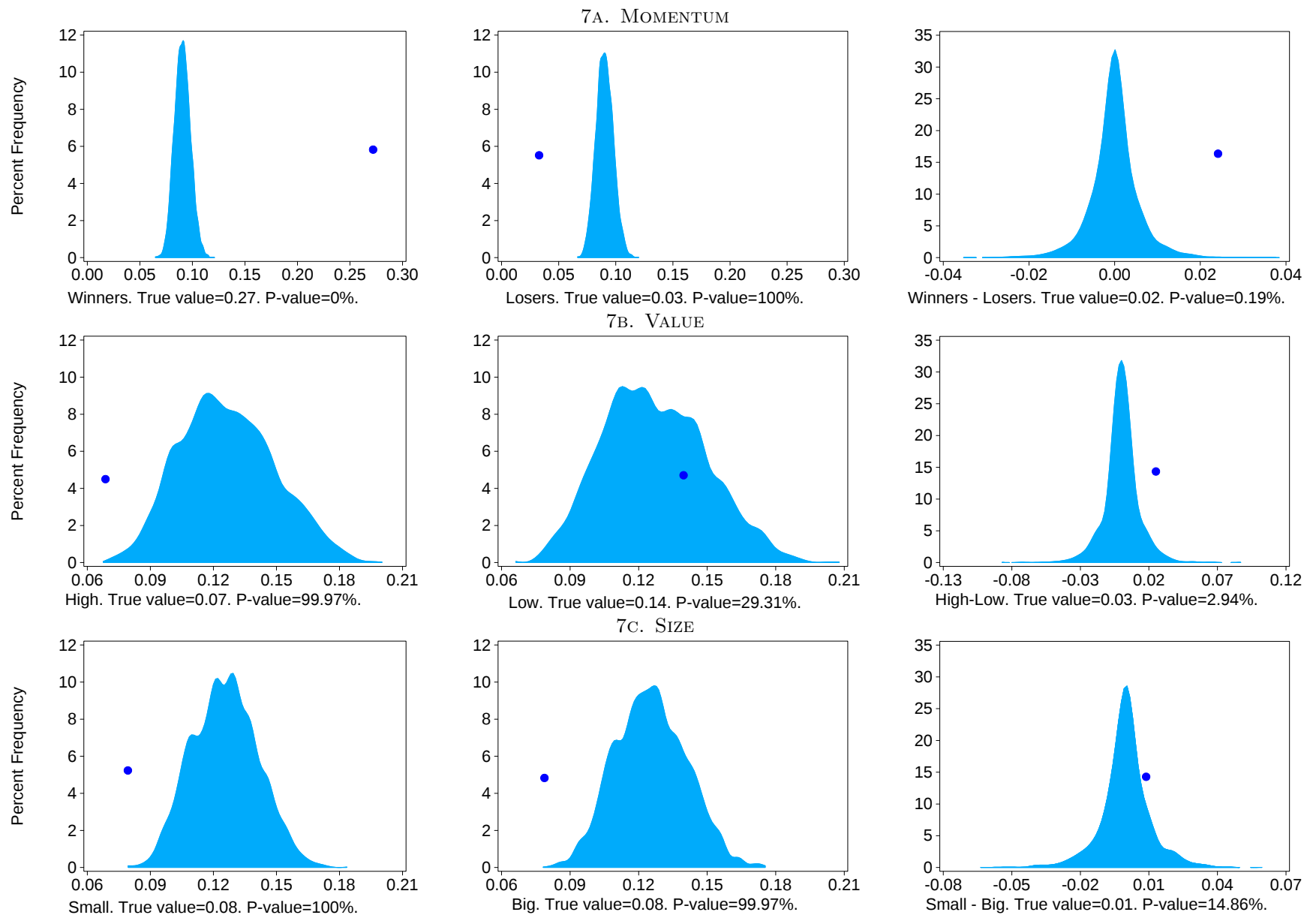


Figure 7: Ratio of Average Return to Kurtosis. This figure plots the empirical distributions of the ratios of average monthly returns, in percent per month, to kurtosis of the randomly selected portfolios. The true ratio of each strategy portfolio is noted in the horizontal axis caption, and is indicated by a dot in each figure. P-value refers to the percent of the random observations that are greater than the true ratio.

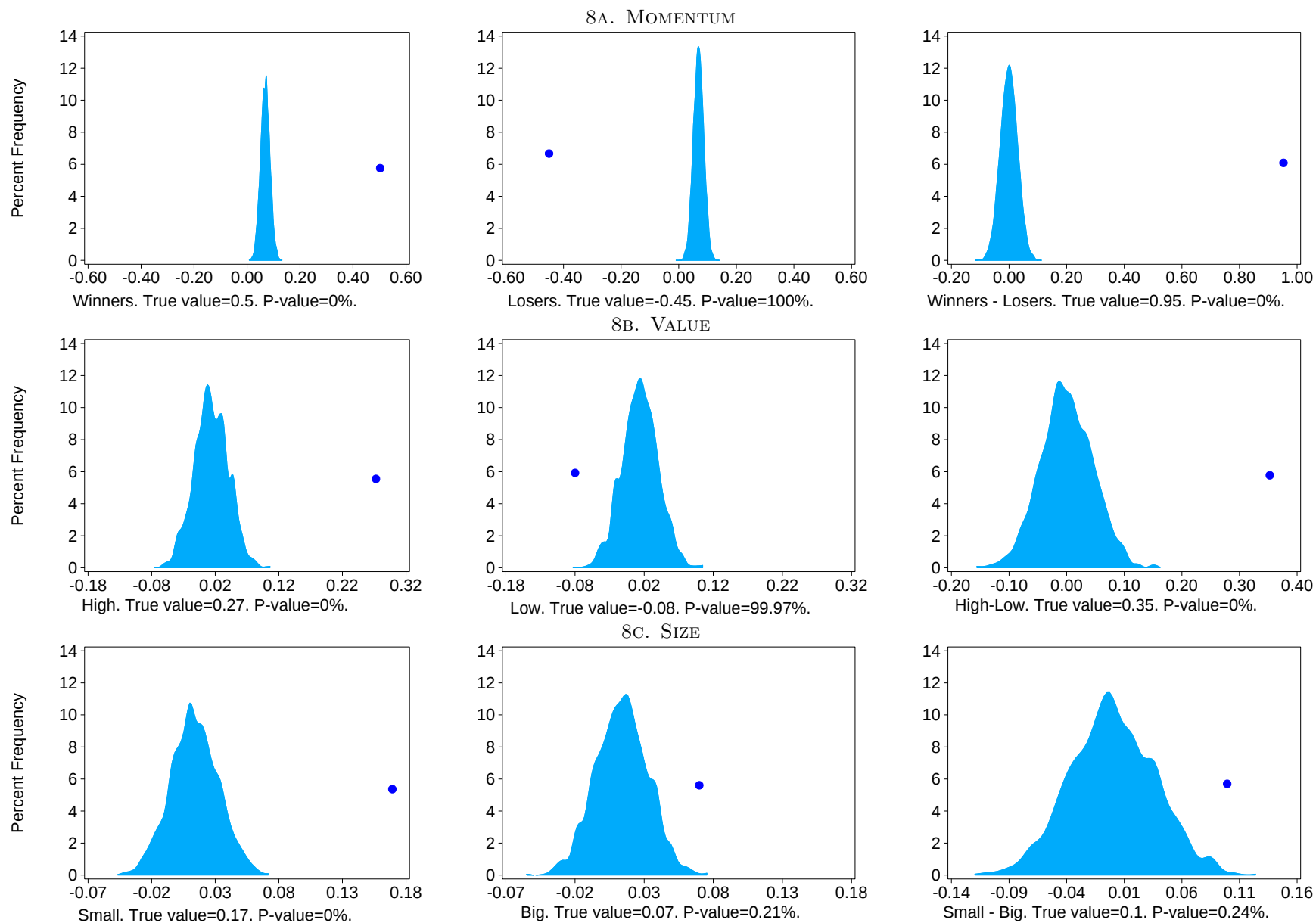


Figure 8: Market Model Alphas. This figure plots the empirical distributions of market model alphas, in percent per month, of the randomly selected portfolios. The true alpha of each strategy portfolio is noted in the horizontal axis caption, and is indicated by a dot in each figure. P-value refers to the percent of the random observations that are greater than the true alpha.

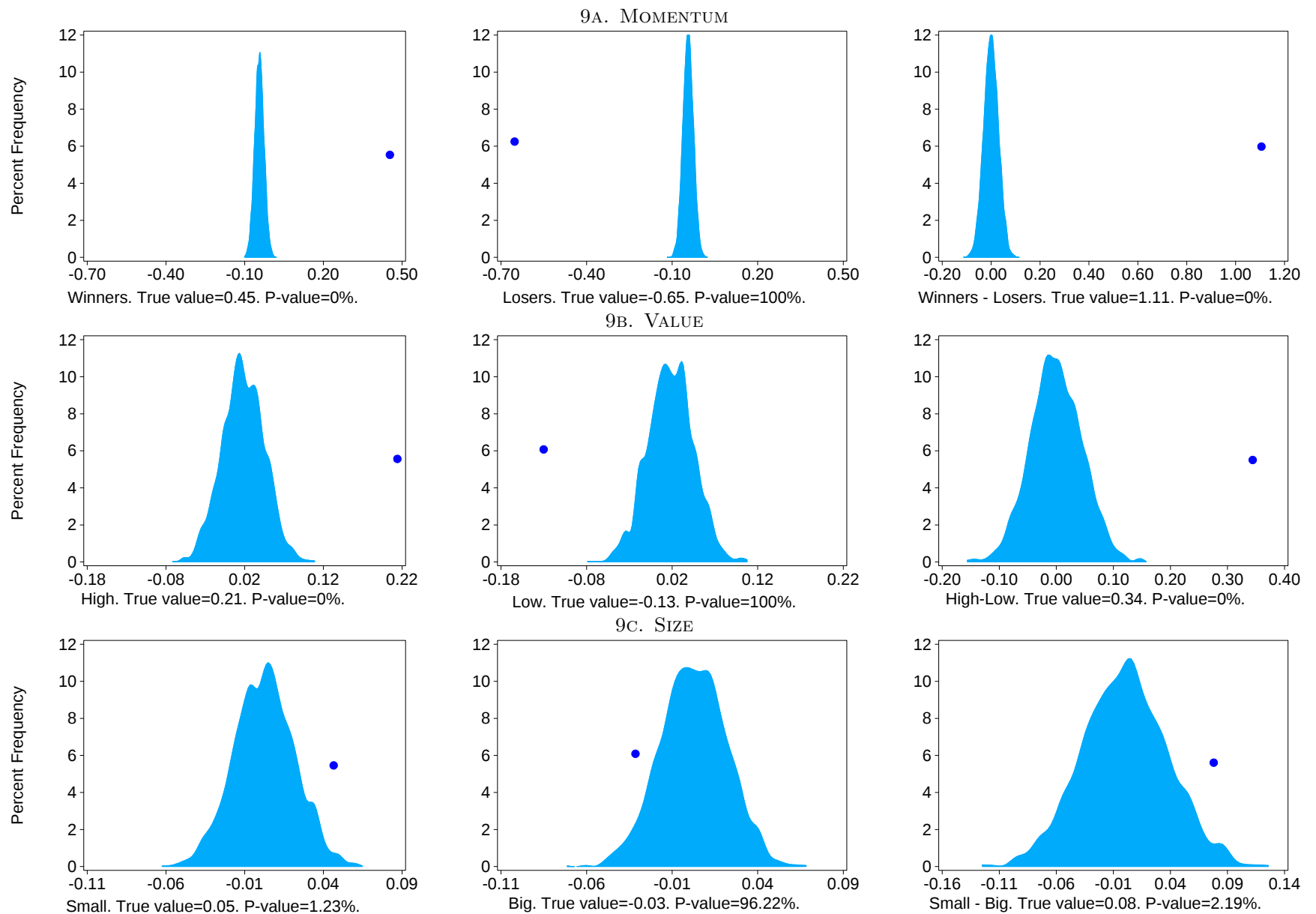


Figure 9: 2- and 3-Factor Alphas. This figure plots the empirical distributions of the 3-factor (for momentum) and 2-factor (for value and size) model alphas of the randomly selected portfolios. The true alpha of each strategy portfolio is noted in the horizontal axis caption, and is indicated by a dot in each figure. P-value refers to the percent of the random observations that are greater than the true alpha.

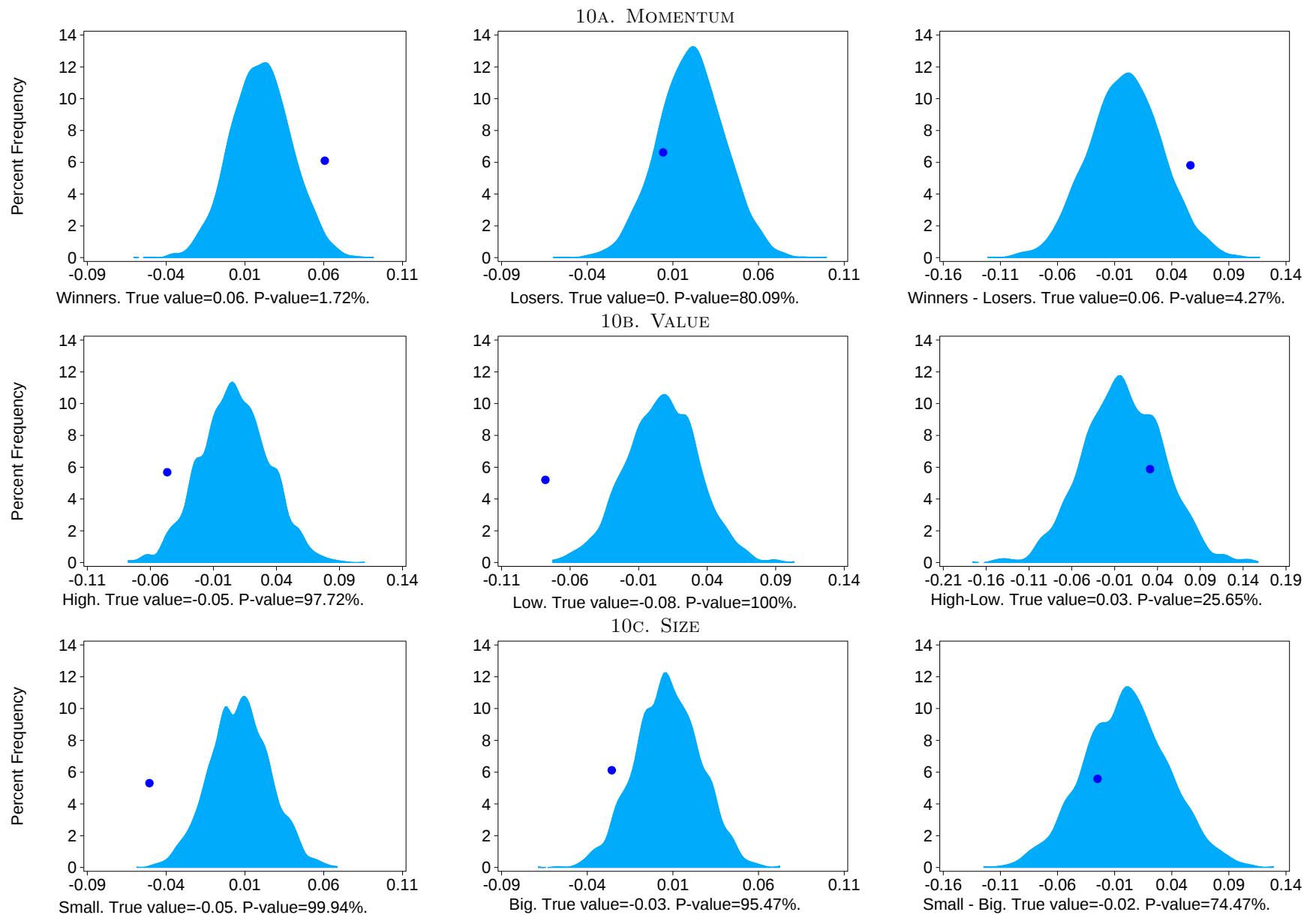


Figure 10: 3- and 4-Factor Alphas. This figure plots the empirical distributions of the 4-factor (for momentum) and 3-factor (for value and size) model alphas of the randomly selected portfolios. The true alpha of each strategy portfolio is noted in the horizontal axis caption, and is indicated by a dot in each figure. P-value refers to the percent of the random observations that are greater than the true alpha.