

Equilibrium Growth, Inflation, and Bond Yields

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Abstract

This paper explores bond pricing implications of a stochastic endogenous growth model with imperfect price adjustment. In this setting, the production and price-setting decisions of firms drive low-frequency movements in macro growth and inflation rates that are negatively related, as in the data. With recursive preferences, these endogenous long-run growth and inflation dynamics are crucial for explaining a number of stylized facts in bond markets. Notably, when calibrated to a wide range of macroeconomic data, the model quantitatively explains the means and volatilities of nominal bond yields. The model also generates a sizeable equity premium and high investment volatility.

JEL Classification: E43, E44, G12, G18

Keywords: Term structure of interest rates, asset pricing, recursive preferences, monetary policy, endogenous growth, inflation, productivity.

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1 Introduction

Explaining the nominal yield curve is a challenge for standard economic models. Notably, Dunn and Singleton (1986) and Backus, Gregory, and Zin (1989) highlight the difficulty of consumption-based models with standard preferences in explaining the sign, magnitude, and volatility of the term spread. More recently, consumption-based models with richer preference specifications and model dynamics, such as Wachter (2006), Piazzesi and Schneider (2006), Gallmeyer, Hollifield, Palomino, and Zin (2007), Bansal and Shaliastovich (2009), have found success in reconciling bond prices. However, Donaldson, Johnsen, and Mehra (1990), den Haan (1995), Rudebusch and Swanson (2008), Palomino (2010), van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2011), and Rudebusch and Swanson (2012), show that extending these endowment economy models to environments with endogenous production still have difficulty in explaining the term premium jointly with key macroeconomic aggregates. This paper shows that endogenizing long-run growth prospects in a production-based model and assuming agents have recursive preferences can help rationalize the slope of the yield curve jointly with a broad spectrum of macroeconomic facts.

Specifically, I link nominal bond prices to firm decisions using a stochastic endogenous growth model with imperfect price adjustment. Inflation is determined by the price-setting behavior of monopolistic firms. Due to the assumption of sticky prices, in equilibrium, inflation is equal to the present discounted value of current and future marginal costs of the firm. Notably, marginal costs are negatively related to productivity, which is crucial for understanding the link between inflation and growth in the model. Long-run productivity growth is driven by R&D investments and leads to sustained growth. In a stochastic setting, this mechanism generates an endogenous stochastic trend and leads to substantial long-run uncertainty about future growth. This framework has two distinguishing features. First, I embed an endogenous growth model of vertical innovations¹ into a standard New Keynesian DSGE model,² which in contrast to the latter type of models, trend growth is endogenously determined by firm investments. Second, households are assumed to have recursive preferences³ so that they are sensitive towards uncertainty about long-term growth

¹See e.g. Grossman and Helpman (1991a), Aghion and Howitt (1992), and Peretto (1999).

²See Woodford (2003) and Galí (2008) for textbook treatments on this class of models.

³See Epstein and Zin (1989) and Weil (1990).

prospects.

While New Keynesian DSGE models have been successful in quantitatively explaining a wide array of empirical macroeconomic features and become the standard framework for modern monetary policy analysis, these models have typically found difficulty in replicating salient features in asset markets.⁴ The results of this paper show that incorporating the endogenous growth margin, in conjunction with recursive preferences, allows these models to reconcile a broad set of asset market facts, including the average nominal yield curve and the equity premium, jointly with macroeconomic facts.

The endogenous growth channel generates persistent low-frequency movements in aggregate growth and inflation rates that are negatively correlated.⁵ The intuition for these dynamics is as follows. In good times, innovation rates increase, which lead to a persistent rise in productivity growth and consequently, a persistent increase in consumption and dividend growth. Furthermore, a prolonged boom in productivity growth lowers marginal cost of firms for an extended period, which leads firms to lower the price of their goods persistently; in the aggregate, this implies a prolonged decline in inflation. When agents have Epstein-Zin recursive utility with a preference for an early resolution of uncertainty, they are very averse to persistent changes in long-run growth prospects. Hence, the innovation-driven low-frequency cycles in consumption and dividend growth imply a high equity premium. In addition, the long-run negative correlation between consumption growth and inflation implies that long nominal bonds have low payoffs when long-run growth is expected to be low. Thus, long bonds are particularly risky which lead to an upward sloping yield curve and sizeable bond risk premium.⁶

When monetary policy follows a Taylor rule and is aggressive in stabilizing inflation, the negative long-run relationship between growth and inflation also implies that an increase in the slope of the

⁴See Smets and Wouters (2007) and Christiano, Eichenbaum, and Evans (2005) are examples of these models that have been able to match the impulse responses of key macroeconomic variables to nominal and real shocks. Rudebusch and Swanson (2008) and Kurmann and Otrok (2011) are examples that document the failures of such models in replicating asset price facts.

⁵There is strong empirical support for these dynamics. For example, Barksy and Sims (2009) and Kurmann and Otrok (2011) document that anticipations of future increases in productivity growth are associated with sharp and persistent declines in inflation.

⁶The basic mechanism that long-run movements in productivity growth drive term spreads is empirically supported. Indeed, Kurmann and Otrok (2010) use a VAR to show that news about long-run productivity growth explain over 60% of the variation in the slope of the term structure.

yield curve predicts higher future growth. For example, suppose that inflation rises. The monetary authority will respond by lowering the short rate. A temporary decline in the short rate implies that future short rates are expected to rise, which steepens the slope of the yield curve. In addition, a fall in inflation is associated with higher expected growth. Thus, a rise in the slope of the yield curve forecasts higher future economic growth, as in the data.⁷

Given that a calibrated version of the model can rationalize a broad set of asset pricing and macroeconomic facts, this framework serves as an ideal laboratory to quantitatively evaluate the effect of changes in monetary policy on asset prices in counterfactual policy experiments. Specifically, in the model, monetary policy is characterized by a short-term nominal interest rate rule that responds to current inflation and output deviations. Due to the presence of nominal rigidities, changes in the short rate affect the real rate which alters real decisions, including R&D. Thus, monetary policy can influence trend growth dynamics. Moreover, by varying the policy parameters, even such short-run stabilization policies can have substantial effects on the level and dynamics of long-run growth, which in turn have important implications for real and nominal risks. For example, more aggressive output stabilization implies that the short rate, and thus the real rate, will increase more in response to an increase in output. Since R&D rates are procyclical, a larger rise in the real rate will dampen the increase in R&D more and lower the volatility of expected growth rates. With less uncertainty about long-run growth prospects, risk premia decline. On the other hand, more aggressive output stabilization amplifies the volatility of expected inflation. Since expected inflation is countercyclical, a sharper rise in the real rate depresses expected inflation even further. Greater uncertainty about expected inflation makes long bonds even riskier which increases the slope of the nominal yield curve.

In another example, more aggressive inflation stabilization implies that the short rate, and thus the real rate, will increase more in response to an increase in inflation. Since inflation and R&D rates are negatively correlated, a larger rise in the real rate will further depress R&D and thus, amplify R&D rates. Furthermore, more volatile R&D rates imply that expected growth rates are more volatile. Higher uncertainty about long-term growth prospects therefore increases risk premia. Additionally, more aggressive inflation stabilization will naturally smooth expected inflation which

⁷See, for example, Ang, Piazzesi, and Wei (2006) for empirical evidence of this relationship.

lowers the slope of the nominal yield curve. Thus, inflation and output stabilization have opposite effects on asset markets. In short, these results suggest that monetary policy, even when targeting short-run deviations, can have a substantial impact on asset markets by distorting long-run growth and inflation dynamics.

My paper is related to a number of different strands of literature in asset pricing, economic growth and macroeconomics. The basic economic mechanisms driving the equity markets are closely related to Bansal and Yaron (2004) (henceforth, BY). In a consumption-based model, BY specify both consumption and dividend growth to contain a small, persistent component, which exogenously leads to long and persistent swings in the dynamics of these quantities. This specification along with the assumption of Epstein-Zin recursive utility allows them to generate high equity premia as compensation for these long-run risks.

The economic mechanisms driving the nominal bond prices are related to the endowment economy models of Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2009). Both of these papers extend the BY framework to a nominal setting by specifying the evolution of inflation exogenously. Critically, in order to match the upward sloping nominal yield curve, both papers require that the long-run correlation between consumption growth and inflation is negative, which they find empirical support for. These joint consumption and inflation dynamics imply that long bonds are particularly risky, as they have low payoffs when expected consumption growth is low. My paper shows that these joint consumption and inflation processes are a natural implication of a stochastic endogenous growth model.

Methodologically, my paper is closely related to Kung and Schmid (2011) (henceforth, KS) who show that in a standard stochastic endogenous growth model with expanding variety, equilibrium R&D decisions generate persistent low-frequency movements in measured productivity growth. Naturally, these productivity dynamics are then reflected in consumption and dividend growth. With recursive preferences, these endogenous low-frequency cycles help to reconcile equity market data. Further, KS documents that the model creates a strong feedback effect between asset prices and growth, which amplifies low-frequency movements in aggregate growth rates, which further increases risk premia. My paper relies on similar economic mechanisms for generating a sizeable equity premium and low riskfree rate. On the other hand, my paper differs from KS by extending

these ideas to a nominal economy with imperfect price adjustment. Specifically, I embed an endogenous growth model of vertical innovations into a standard New Keynesian setup. These extensions allow me to study the determination of nominal bond prices jointly with equity prices and also, to analyze the implications of monetary policy for both growth and asset prices.

More broadly, my paper relates to a number of recent papers that study how long-run risks arise endogenously in general equilibrium production economies. Some examples include Tallarini (2000), Uhlig (2010), Backus, Routledge, and Zin (2010), Croce (2008), Campanale, Castro, and Clementi (2008), Kaltenbrunner and Lochstoer (2008), Kuehn (2008), Ai (2009), Gourio (2009), and Kuehn, Petrosky-Nadeau, and Zhang (2011). These papers typically work in versions of the standard real business cycle model, where growth is given exogenously. One conclusion from calibrated versions of these important contributions is that while long-run risks do arise endogenously in such settings, they are typically not quantitatively sufficient to rationalize key asset market statistics. In contrast, the endogenous growth paradigm does deliver quantitatively significant endogenous long-run risks through the R&D and innovation decisions of firms.

My paper also relates to the literature examining the term structure of interest rates in general equilibrium production-based models.⁸ Donaldson, Johnsen, and Mehra (1990) and den Haan (1995) document that variants of standard business cycle models with additively separable preferences have trouble reproducing the sizeable positive nominal term premium observed in the data. These shortcomings are inherently linked to the fact that additively separable preferences cannot generate sufficient risk premia. Indeed, Backus, Gregory, and Zin (1989) also highlight similar issues in an endowment economy setting.

Wachter (2006) shows that a consumption-based model with habit preferences can explain the nominal term structure of interest rates. However, Rudebusch and Swanson (2008) show that in production-based model with habit preferences, nominal bond prices can only be reconciled at the expense of distorting the fit of macroeconomic variables, such as real wages and inflation. In contrast, my model can explain the nominal structure of interest rates, jointly with equity prices, in a production setting while still maintaining a good fit to a broad set of macroeconomic variables.

van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2011) and Rudebusch and

⁸See Jermann (2011) for an example of a partial equilibrium production-based model of the term structure.

Swanson (2012) consider standard production-based models with recursive preferences and highlight the difficulty these models have in quantitatively explaining the nominal term structure of interest rates with macro aggregates. In particular, these papers demonstrate that a very large coefficient of relative risk aversion is required to match to the slope of the nominal yield curve. In contrast, in my model, I explain the slope of the yield curve with a standard calibration. This difference is again attributed to the fact that standard neoclassical models with exogenous growth lack the strong propagation mechanism of endogenous growth models and therefore do not generate enough long-run consumption growth volatility. So, while the neoclassical models generate negative correlation between consumption growth and inflation to give a positively sloping yield curve, the quantity of real long-run risks is too small; and thus, the bond risk premium is too small.

Furthermore, Rudebusch and Swanson (2012) also document that incorporating expected growth shocks to the productivity process actually makes the nominal yield curve downward sloping. A positive expected productivity growth shock leads to a very large wealth effect that reduces the incentives to work and leads to a sharp rise in wages. The increase in wages raises marginal costs and thus, inflation increases. In addition, expected consumption growth naturally inherits the long-run dynamics of productivity growth. Thus, this shock leads to a positive correlation between expected consumption growth and inflation, which leads to a downward sloping average nominal yield curve. In contrast, the movements in expected productivity growth in my model are endogenous and affected by labor decisions. Namely, an increase in the labor input raises the marginal productivity of R&D and therefore raises the incentives to innovate. An increase in labor hours will raise both the level of output and the expected growth rate of output, *ceteris paribus*. Put differently, the labor input has a significantly higher marginal value in the endogenous growth model than in the neoclassical setting, where productivity is exogenous. Consequently, in the endogenous growth setting, agents have higher incentives to supply labor in good times to boost expected growth prospects. Importantly, this dampens the sharp rise in wages from the wealth effect of persistently higher future growth. Consequently, the endogenous growth channel allows my model to maintain the strong negative correlation between expected consumption growth and inflation that is critical for explaining the nominal yield curve.

Finally, the policy implications of my paper are related to a few recent papers exploring how

various policy instruments can distort the intertemporal distribution of consumption risk. Croce, Kung, Nguyen, and Schmid (2012) (henceforth, CKNS) demonstrate how tax smoothing fiscal policies can amplify low-frequency movements in growth rate in a real business cycle model with financial frictions, which can increase risk premia significantly. Croce, Nyugen, and Schmid (2011) (henceforth, CNS) study fiscal policy design in a stochastic endogenous growth model with expanding variety. Similarly, they find that fiscal policies aimed at short-run stabilization significantly amplify long-run consumption volatility and decreases welfare. In contrast, in my paper, I find that interest rate rules targeting short-run output stabilization decreases long-run consumption volatility while inflation stabilization increases long-run consumption volatility. Thus, while CKNS and CNS study the role of fiscal policy on growth dynamics, my paper studies the role of monetary policy. Hence, I view these papers as complementary.

The paper is structured as follows. Section 2 outlines the benchmark endogenous growth model and the exogenous growth counterparts. Section 3 qualitatively illustrates the growth and inflation dynamics of the model. Section 4 explores the quantitative implications of the model. Section 5 concludes.

2 Model

The benchmark model embeds a endogenous growth model of vertical innovations into a fairly standard New Keynesian model. The representative household is assumed to have recursive preferences defined over consumption and leisure. These preferences imply that the household is sensitive towards fluctuations to expected growth rates, which is a key margin in this model. The production side is comprised of a final goods sector and an intermediate goods sector. The final goods sector is characterized by a representative firm that produces the consumption goods using a bundle of intermediate goods inputs that are purchased from intermediate goods producers. The intermediate goods sector is comprised of a continuum of monopolists of unit measure. Each monopolist sets prices subject to quadratic price adjustment costs and uses firm-specific labor, physical capital, and R&D capital inputs to produce a particular intermediate goods. Also, each monopolist accumulates the physical and R&D capital stocks subject to convex adjustment costs. The monetary authority

is assumed to follow a modified Taylor rule.

Under a certain parameter configuration and exogenous R&D policy, the benchmark endogenous growth model collapses to a standard New Keynesian setup with exogenous growth. Moreover, to highlight the implications of the endogenous growth mechanism, I compare the benchmark growth model to two paradigms of exogenous growth, one with a deterministic trend and the other with a stochastic trend.

2.1 Endogenous Growth

Representative Household Assume that the household has recursive utility over streams of consumption C_t and leisure $\bar{L} - L_t$:

$$\begin{aligned} U_t &= \left\{ (1 - \beta)(C_t^*)^{\frac{1-\gamma}{\theta}} + \beta \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \\ C_t^* &= C_t(\bar{L} - L_t)^\tau \end{aligned}$$

where γ is the coefficient of risk aversion, ψ is the elasticity of intertemporal substitution,⁹ $\theta \equiv \frac{1-\gamma}{1-1/\psi}$ is a parameter defined for convenience, β is the subjective discount rate, and $\bar{L}$ is the agent's time endowment. The time t budget constraint of the household is

$$\mathcal{P}_t C_t + \frac{\mathcal{B}_{t+1}}{\mathcal{R}_{t+1}} = \mathcal{D}_t + \mathcal{W}_t L_t + \mathcal{B}_t$$

where $\mathcal{P}_t$ is the nominal price of the final goods, $\mathcal{B}_{t+1}$ are nominal one-period bonds, $\mathcal{R}_{t+1}$ is the gross nominal interest rate set at time t by the monetary authority, $\mathcal{D}_t$ is nominal dividend income received from the intermediate firms, $\mathcal{W}_t$ is the nominal wage rate, and L_t is labor supplied by the household. The household's intertemporal condition is

$$1 = E_t \left[M_{t+1} \frac{\mathcal{P}_t}{\mathcal{P}_{t+1}} \right] \mathcal{R}_{t+1}$$

⁹The parameters γ and ψ are defined over the composite good C_t^* .

where

$$M_{t+1} = \beta \left(\frac{C_{t+1}^*}{C_t^*} \right)^{\frac{1-\gamma}{\theta}} \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left(\frac{U_{t+1}^{1-\gamma}}{E_t[U_{t+1}^{1-\gamma}]} \right)^{1-\frac{1}{\theta}}$$

is the stochastic discount factor. The intratemporal condition is

$$\frac{\mathcal{W}_t}{\mathcal{P}_t} = \frac{\tau C_t}{\bar{L} - L_t}$$

Final Goods A representative firm produces the final (consumption) goods in a perfectly competitive market. The firm uses a continuum of differentiated intermediate goods $X_{i,t}$ as input in the CES production technology

$$Y_t = \left(\int_0^1 X_{i,t}^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}}$$

where ν is the elasticity of substitution between intermediate goods. The profit maximization problem of the firm yields the following isoelastic demand schedule with price elasticity ν

$$X_{i,t} = Y_t \left(\frac{\mathcal{P}_{i,t}}{\mathcal{P}_t} \right)^{-\nu}$$

where $\mathcal{P}_t$ is the nominal price of the final goods and $\mathcal{P}_{i,t}$ is the nominal price of intermediate goods i . The inverse demand schedule is

$$\mathcal{P}_{i,t} = \mathcal{P}_t Y_t^{\frac{1}{\nu}} X_{i,t}^{-\frac{1}{\nu}}$$

Intermediate Goods The intermediate goods sector will be characterized by a continuum of monopolistic firms. Each intermediate goods firm produces $X_{i,t}$ with physical capital $K_{i,t}$, R&D capital $N_{i,t}$, and labor $L_{i,t}$ inputs using the following technology, similar to Peretto (1999),

$$X_{i,t} = K_{i,t}^\alpha (Z_{i,t} L_{i,t})^{1-\alpha}$$

where total factor productivity (TFP) is

$$Z_{i,t} \equiv A_t N_{i,t}^\eta N_t^{1-\eta}$$

where A_t represents a stationary aggregate productivity shock, $N_t \equiv \int_0^1 N_j dj$ is the aggregate stock of R&D and the parameter $\eta \in [0, 1]$ captures the degree of technological appropriability. Thus, firm-level TFP is comprised of two aggregate components, A_t and N_t , and a firm-specific component $N_{i,t}$. In contrast to the neoclassical production function with labor augmenting technology, TFP contains an endogenous component determined by firm decisions. In particular, the firm can upgrade its technology directly by investing in R&D. Furthermore, there are spillover effects from innovating; namely, firm-level investments in R&D will also improve aggregate technology. These spillover effects are crucial for generating sustained growth in the economy and are a standard feature in modern endogenous growth models.¹⁰

The law of motion for A_t , in logs, is

$$a_t = (1 - \rho)a^* + \rho a_{t-1} + \sigma \epsilon_t$$

where $a_t \equiv \log(A_t)$, $\epsilon_t \sim N(0, 1)$ is i.i.d., and $a^* > 0$ is the unconditional mean of a_t .

The law of motion for $K_{i,t}$ is

$$\begin{aligned} K_{i,t+1} &= (1 - \delta_k)K_{i,t} + \Phi_k\left(\frac{I_{i,t}}{K_{i,t}}\right) K_{i,t} \\ \Phi_k\left(\frac{I_{i,t}}{K_{i,t}}\right) &= \frac{\alpha_{1,k}}{1 - \frac{1}{\zeta_k}} \left(\frac{I_{i,t}}{K_{i,t}}\right)^{1 - \frac{1}{\zeta_k}} + \alpha_{2,k} \end{aligned}$$

where $I_{i,t}$ is capital investment (using the final goods) and the function $\Phi_k(\cdot)$ captures capital adjustment costs.¹¹ The parameter ζ_k represents the elasticity of new capital investments relative to the existing stock of capital. The parameters $\alpha_{1,k}$ and $\alpha_{2,k}$ are set to values so that there are no adjustment costs in the deterministic steady state.¹²

¹⁰See, for example, Romer (1990), Grossman and Helpman (1991b), and Aghion and Howitt (1992).

¹¹See, for example, Jermann (1998) and Zhang (2005) for the importance of adjustment costs for asset prices.

¹²Specifically, $\alpha_{1,k} = (\Delta Z_{ss} - 1 + \delta_k)^{\frac{1}{\zeta_k}}$ and $\alpha_{2,k} = \frac{1}{\zeta_k - 1}(1 - \delta_k - \Delta Z_{ss})$.

The law of motion for $N_{i,t}$ is

$$\begin{aligned} N_{i,t+1} &= (1 - \delta_n)N_{i,t} + \Phi_n \left(\frac{S_{i,t}}{N_{i,t}} \right) N_{i,t} \\ \Phi_n \left(\frac{S_{i,t}}{N_{i,t}} \right) &= \frac{\alpha_{1,n}}{1 - \frac{1}{\zeta_n}} \left(\frac{S_{i,t}}{N_{i,t}} \right)^{1 - \frac{1}{\zeta_n}} + \alpha_{2,n} \end{aligned}$$

where $S_{i,t}$ is R&D investment (using the final goods) and the function $\Phi_n(\cdot)$ captures adjustment costs in R&D investments. The parameter ζ_n represents the elasticity of new R&D investments relative to the existing stock of R&D. The parameters $\alpha_{1,n}$ and $\alpha_{2,n}$ are set to values so that there are no adjustment costs in the deterministic steady state.¹³

Substituting the production technology into the inverse demand function yields the following expression for the nominal price for intermediate goods i

$$\begin{aligned} \mathcal{P}_{i,t} &= \mathcal{P}_t Y_t^{\frac{1}{\nu}} \left[K_{i,t}^\alpha \left(A_t N_{i,t}^\eta N_t^{1-\eta} L_{i,t} \right)^{1-\alpha} \right]^{-\frac{1}{\nu}} \\ &\equiv \mathcal{P}_t J(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t) \end{aligned}$$

Further, nominal revenues for intermediate firm i can be expressed as

$$\begin{aligned} \mathcal{P}_{i,t} X_{i,t} &= \mathcal{P}_t Y_t^{\frac{1}{\nu}} \left[K_{i,t}^\alpha \left(A_t N_{i,t}^\eta N_t^{1-\eta} L_{i,t} \right)^{1-\alpha} \right]^{1 - \frac{1}{\nu}} \\ &\equiv \mathcal{P}_t F(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t) \end{aligned}$$

For the real revenue function $F(\cdot)$ to exhibit diminishing returns to scale in the factors $K_{i,t}$, $L_{i,t}$, and $N_{i,t}$ requires the following parameter restriction:

$$[\alpha + (\eta + 1)(1 - \alpha)] \left(1 - \frac{1}{\nu} \right) < 1$$

or

$$\eta(1 - \alpha)(\nu - 1) < 1$$

¹³Specifically, $\alpha_{1,n} = (\Delta N_{ss} - 1 + \delta_n)^{\frac{1}{\zeta_n}}$ and $\alpha_{2,n} = \frac{1}{\zeta_n - 1}(1 - \delta_n - \Delta N_{ss})$.

The intermediate firms face a cost of adjusting the nominal price à la Rotemberg (1982), measured in terms of the final goods as

$$G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}; \mathcal{P}_t, Y_t) = \frac{\phi_R}{2} \left(\frac{\mathcal{P}_{i,t}}{\Pi_{ss} \mathcal{P}_{i,t-1}} - 1 \right)^2 Y_t$$

where $\Pi_{ss} \geq 1$ is the gross steady-state inflation rate and ϕ_R is the magnitude of the costs. The source of funds constraint is

$$\mathcal{D}_{i,t} = \mathcal{P}_t F(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t) - \mathcal{W}_{i,t} L_{i,t} - \mathcal{P}_t I_{i,t} - \mathcal{P}_t S_{i,t} - \mathcal{P}_t G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}; \mathcal{P}_t, Y_t)$$

where $\mathcal{D}_{i,t}$ and $\mathcal{W}_{i,t}$ are the nominal dividend and wage rate, respectively, for intermediate firm i . Firm i takes the pricing kernel M_t and the vector of aggregate states $\mathbf{\Upsilon}_t = [\mathcal{P}_t, K_t, N_t, Y_t, A_t]$ as given and solves the following recursive program to maximize shareholder value $V_{i,t} \equiv V^{(i)}(\cdot)$

$$V^{(i)}(\mathcal{P}_{i,t-1}, K_{i,t}, N_{i,t}; \mathbf{\Upsilon}_t) = \max_{\mathcal{P}_{i,t}, I_{i,t}, S_{i,t}, K_{i,t+1}, N_{i,t+1}, L_{i,t}} \frac{\mathcal{D}_{i,t}}{\mathcal{P}_t} + E_t \left[M_{t+1} V^{(i)}(\mathcal{P}_{i,t}, K_{i,t+1}, N_{i,t+1}; \mathbf{\Upsilon}_{t+1}) \right]$$

subject to

$$\begin{aligned} \frac{\mathcal{P}_{i,t}}{\mathcal{P}_t} &= J(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t) \\ K_{i,t+1} &= (1 - \delta_k) K_{i,t} + \Phi_k \left(\frac{I_{i,t}}{K_{i,t}} \right) K_{i,t} \\ N_{i,t+1} &= (1 - \delta_n) N_{i,t} + \Phi_n \left(\frac{S_{i,t}}{N_{i,t}} \right) N_{i,t} \\ \mathcal{D}_{i,t} &= \mathcal{P}_t F(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t) - \mathcal{W}_{i,t} L_{i,t} - \mathcal{P}_t I_{i,t} - \mathcal{P}_t S_{i,t} - \mathcal{P}_t G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}; \mathcal{P}_t, Y_t) \end{aligned}$$

The corresponding first-order conditions can be found in Appendix B.

Central Bank The central bank follows a modified Taylor rule specification that depends on the lagged interest rate and output and inflation deviations:

$$\ln \left(\frac{\mathcal{R}_{t+1}}{\mathcal{R}_{ss}} \right) = \rho_r \ln \left(\frac{\mathcal{R}_t}{\mathcal{R}_{ss}} \right) + (1 - \rho_r) \left(\rho_\pi \ln \left(\frac{\Pi_t}{\Pi_{ss}} \right) + \rho_y \ln \left(\frac{\hat{Y}_t}{\hat{Y}_{ss}} \right) \right) + \sigma_\xi \xi_t$$

where $\mathcal{R}_{t+1}$ is the gross nominal short rate, $\hat{Y}_t \equiv \frac{Y_t}{N_t}$ is detrended output, and $\xi_t \sim N(0, 1)$ is a monetary policy shock. Variables with a *ss*-subscript denote steady-state values. Given this rule, the central bank chooses ρ_r , ρ_π , ρ_y , and Π_{ss} .

Symmetric Equilibrium In the symmetric equilibrium, all intermediate firms make identical decisions: $\mathcal{P}_{i,t} = \mathcal{P}_t$, $X_{i,t} = X_t$, $K_{i,t} = K_t$, $L_{i,t} = L_t$, $N_{i,t} = N_t$, $I_{i,t} = I_t$, $S_{i,t} = S_t$, $D_{i,t} = D_t$, $V_{i,t} = V_t$. Also, $\mathcal{B}_t = 0$. The aggregate resource constraint is

$$Y_t = C_t + S_t + I_t + \frac{\phi_R}{2} \left(\frac{\Pi_t}{\Pi_{ss}} - 1 \right)^2 Y_t$$

where $\Pi_t \equiv \frac{\mathcal{P}_t}{\mathcal{P}_{t-1}}$ is the gross inflation rate.

Nominal Yields The price of a n -period nominal bond $\mathcal{P}_t^{(n)}$ can be written recursively as:

$$\mathcal{P}_t^{(n)} = E_t \left[M_{t+1} \frac{1}{\Pi_{t+1}} \mathcal{P}_{t+1}^{(n-1)} \right]$$

where $\mathcal{P}_t^{(0)} = 1$ and $\mathcal{P}_t^{(1)} = \frac{1}{\mathcal{R}_{t+1}}$. The yield-to-maturity on the n -period nominal bond is defined as:

$$y_t^{(n)} \equiv -\frac{1}{n} \log \left(\mathcal{P}_t^{(n)} \right)$$

2.2 Exogenous Growth

This setup also nests a fairly standard New Keynesian setup with exogenous growth when the technological appropriability parameter η is set to 0 and the aggregate stock of R&D N_t is exogenously specified. Under these conditions the production function of the intermediate firm can be expressed as

$$\begin{aligned} X_{i,t} &= K_{i,t}^\alpha (Z_t L_{i,t})^{1-\alpha} \\ Z_t &= A_t N_t \end{aligned}$$

where N_t follows a stochastic process. Note that TFP is now exogenous and comprised of a stationary component A_t and a trend component N_t . I consider two versions of the exogenous growth model, one with a deterministic trend and the other with a stochastic trend in productivity, to compare with the benchmark endogenous growth model.

Deterministic Trend The law of motion for N_t is

$$N_t = e^{\mu t}$$

where μ is parameter governing the average growth rate of the economy. Equivalently, this expression can be rewritten in log first differences as

$$\Delta n_t = \mu$$

where $\Delta n_t \equiv \ln(N_t) - \ln(N_{t-1})$.

Stochastic Trend The log growth rate of N_t is specified as

$$\Delta n_t = \mu + x_{t-1}$$

$$x_t = \rho_x x_{t-1} + \sigma_x \epsilon_{x,t}$$

where $\epsilon_{x,t} \sim iid N(0, 1)$, $corr(\epsilon_t, \epsilon_{x,t}) = 0$, and ρ_x is the persistence parameter of the autoregressive process x_t .

3 Equilibrium Growth and Inflation

This section will provide a qualitative description of the growth and inflation dynamics. Sustained growth in the benchmark endogenous growth model is attributed to the R&D decisions of intermediate firms. In a stochastic setting, this framework generates (i) low-frequency movements in the growth rate of real and nominal variables and (ii) a negative long-run correlation between aggregate growth rates and inflation. These long-run dynamics are essential for explaining asset market

data. In contrast, a standard model with exogenous growth lacks a strong propagation mechanism that can generate low-frequency movements in growth rates. While incorporating exogenous low-frequency shocks into productivity growth in the neoclassical model will lead to low-frequency cycles in aggregate growth rates, this shock will consequently drive a strong positive relation between expected consumption growth and expected inflation. Furthermore, the endogenous growth paradigm allows monetary policy to play an important role in influencing long-run growth dynamics, and hence risk premia. This section describes the equilibrium growth and inflation dynamics qualitatively, and contrast them with the exogenous growth setups described above. The next section presents empirical evidence supporting these patterns and provides a quantitative assessment of the model.

Growth Dynamics First, I will characterize the equilibrium growth dynamics. In the benchmark endogenous growth model, substituting in the symmetric equilibrium conditions yields the following the aggregate production function

$$\begin{aligned} Y_t &= K_t^\alpha (Z_t L_t)^{1-\alpha} \\ Z_t &\equiv A_t N_t \end{aligned}$$

Recall that the trend component, N_t , is the equilibrium stock of aggregate R&D capital in this setting and endogenously determined by the intermediate firms. In other words, the benchmark model generates an endogenous stochastic trend. Furthermore, log productivity growth is

$$\Delta z_t = \Delta a_t + \Delta n_t$$

Since, a_t is typically calibrated to be persistent shock, then $\Delta a_t \approx \epsilon_t$. Using this approximation we can rewrite the expression above as

$$\Delta z_t = \Delta n_t + \epsilon_t$$

Also, since Δn_t is determined at $t - 1$, the expected growth rate is equal to the stock of R&D growth:

$$E_{t-1}[\Delta z_t] = \Delta n_t$$

Thus, low-frequency movements are driven by endogenous R&D rates, which are fairly persistent and volatile processes in the data. Furthermore, the propagation mechanism of endogenous growth framework implies that the single shock a_t will generate both high-frequency movements and low-frequency movements. In contrast, in a standard neoclassical paradigm with a deterministic trend, log productivity growth is

$$\Delta z_t = \mu + \epsilon_t$$

where again, the approximation $\Delta a_t \approx \epsilon_t$ is used. Hence, the expected growth rate is constant:

$$E_{t-1}[\Delta z_t] = \mu$$

Hence, the exogenous growth model will only exhibit high-frequency movements around the trend.

In the exogenous growth model with a stochastic trend, productivity growth is

$$\Delta z_t = \mu + x_{t-1} + \epsilon_t$$

where again, the approximation $\Delta a_t \approx \epsilon_t$ is used. Thus the expected growth rate is equal to the drift plus a persistent shock x_{t-1} , often referred to in the literature as “long-run productivity risk”¹⁴

$$E_{t-1}[\Delta z_t] = \mu + x_{t-1}$$

Thus, the endogenous growth paradigm provides a structural interpretation for this low-frequency component by linking it to innovation rates.¹⁵ Importantly, in the endogenous growth framework,

¹⁴See Croce (2010).

¹⁵Kung and Schmid (2011) also highlight this mechanism in explaining the equity premium.

the monetary authority can affect this low-frequency component, or in other words, alter the distribution of long-run productivity risk. As will be shown in the next section, monetary policy can significantly alter risk premia through this channel.

Inflation Dynamics In the models above, the log-linearized inflation dynamics depend on real marginal costs and expected inflation:

$$\tilde{\pi}_t = \gamma_1 \tilde{mc}_t + \gamma_2 E_t[\tilde{\pi}_{t+1}]$$

where $\gamma_1 = \frac{\nu-1}{\phi_R} > 0$, $\gamma_2 = \beta \Delta Y_{ss}^{1-\frac{1}{\psi}} > 0$, and lowercase tilde variables denote log deviations from the steady-state.¹⁶ Recursively substituting out future $\tilde{\pi}$ terms implies that current and expected future real marginal costs drive inflation dynamics. Moreover, real marginal cost can be expressed as the ratio between real wages and the marginal product of labor. Furthermore, real marginal costs, in log-linearized form, can be expressed as:

$$\tilde{mc}_t = \tilde{w}_t + \alpha \tilde{l}_t - (1 - \alpha) \tilde{a}_t - (1 - \alpha) \tilde{n}_t$$

where lowercase tilde variables denote log deviations from the steady-state. Thus, inflation is driven by the relative dynamics of these aggregate variables. In the endogenous growth model, after a good productivity shock, $\tilde{w}_t$, $\tilde{l}_t$, and $\tilde{n}_t$ all increase after an increase in $\tilde{a}_t$. Notably, in a calibrated version of the benchmark model, the magnitude of the responses of last two terms in the equation are larger than that of the first two terms. Consequently, marginal costs and inflation decrease persistently after a positive productivity shock. On the other hand, as discussed above, expected growth rates increase persistently after a positive productivity shock. Thus, expected inflation and expected growth rates have a strong negative relationship in the benchmark model, as in the data. These dynamics will be examined further in the section below.

¹⁶See Appendix C for details.

4 Quantitative Results

This section explores the quantitative implications of the model using simulations. Perturbation methods are used to solve the model. To account for risk premia and potential time variation, a higher-order approximation around the stochastic steady state is used. Furthermore, ENDO 1 will refer to the benchmark endogenous growth model, ENDO 2 is the same as ENDO 1 but with no policy uncertainty, EXO 1 will refer to the exogenous growth model with a deterministic trend, and EXO 2 will refer to the exogenous growth model with a stochastic trend.

4.1 Calibration

This part presents the quarterly calibration used to assess the quantitative implications of the benchmark growth model (ENDO 1). Table 1 reports the calibration of the benchmark model along with the other three models that are used for comparison. Worth emphasizing, the core set of results are robust to reasonable deviations around the benchmark calibration. Recursive preferences have been used extensively in recent work in asset pricing.¹⁷ I follow this literature and set preference parameters to standard values that are also supported empirically.¹⁸ Parameters in the final goods and intermediate goods sector are set to standard values in the New Keynesian DSGE literature. Non-standard parameters in the intermediate goods sector are used to match R&D dynamics. Critically, satisfying balanced growth helps provide further restrictions on parameter values.

I begin with a description of the calibration of the preference parameters. The parameter τ is set to match the steady-state household hours worked. The elasticity of intertemporal substitution ψ is set to value of 2 and the coefficient of relative risk aversion γ is set to a value of 10, which are standard values in the long-run risks literature. An intertemporal elasticity of substitution larger than one is consistent with the notion that an increase in uncertainty lowers the price-dividend ratio. Note that in this parametrization, $\psi > \frac{1}{\gamma}$, which implies that the agent dislikes shocks to expected growth rates and is particularly important for generating a sizeable risk premium in this setting. The subjective discount factor β is set to a value of 0.9965 so as to be broadly consistent

¹⁷See Bansal and Yaron (2004).

¹⁸See Bansal, Kiku, and Yaron (2007) uses Euler conditions and a GMM estimator to provide empirical support for the parameter values.

with the level of the riskfree rate. In the endogenous growth setting β also has important effect on the level of the growth rate. In particular, increasing β (the agent is more patient) increases the steady-state growth rate. Holding all else constant (including β), the direct effect of an increase in growth is an increase in the level of the riskfree rate. On the other hand, the direct effect of increasing β and holding the level of the growth rate fixed is a decrease in the level of the riskfree rate.

I now move to the calibration of the standard parameters from the production-side. In the final goods sector, the price elasticity of demand is set at 6, which corresponds to a markup of 0.2. In the intermediate goods sector, the capital share α is set to 0.33 and the depreciation rate of capital δ is set to 0.02, which are calibrated to match steady-state evidence. The quadratic price adjustment function was first proposed in Rotemberg (1982) and is standard in the literature. The price adjustment cost parameter ϕ_R is set to 30, and is calibrated to match the impulse response of output to a monetary policy shock. Furthermore, in a log-linear approximation, the parameter ϕ_R can be mapped directly to a parameter that governs the average price duration in a Calvo pricing framework.¹⁹ In this calibration, $\phi_R = 30$ corresponds to an average price duration of 3 months, which is consistent with micro evidence from Bils and Klenow (2004). The capital adjustment cost function is standard in the production-based asset pricing literature.²⁰ The capital adjustment cost parameter ζ_k is set at 4.7 to match the relative volatility of investment growth to consumption growth.

The nonstandard parameters are now discussed. The depreciation rate of the R&D capital stock δ_n is calibrated to a value of 0.0375 which corresponds to an annualized depreciation rate of 15% which is a standard value and that assumed by the BLS in the the R&D stock calculations. The R&D capital adjustment cost parameter ζ_n is set at 3.3 to match the relative volatility of R&D investment growth to consumption growth. The degree of technological appropriability η is set at 0.1 to match the steady-state value of the R&D investment rate.

I now turn to the calibration of the parameters relating to the stationary productivity shock a_t . Note that this shock is different than the Solow residual since measured productivity includes an

¹⁹See Appendix C for details.

²⁰See, for example, Jermann (1998), Croce (2008), Kaltenbrunner and Lochstoer (2008) or van Binsbergen, Fernandez-Villaverde, Koijen, and Rubio-Ramirez (2011) for estimation evidence.

endogenous component that is related to the equilibrium stock of R&D. A decomposition of total factor productivity in our benchmark model is provided below, which provides a mapping between the exogenous growth model and the endogenous growth model. The persistence parameter ρ is set to 0.985 and is calibrated to match the first autocorrelation of R&D intensity, which determines the growth rate of the R&D stock (expected growth component) and in turn, is a critical determinant of asset prices.²¹ The volatility parameter σ is set at 1.36% to match consumption growth volatility. The constant determining the mean of the process a^* is set to match balanced growth evidence.

The monetary policy rule parameters are within the standard range of estimated values in the literature.²² The parameter governing the sensitivity of the interest rate to inflation deviations ρ_π is set to 1.5. The parameter governing the sensitivity of the interest rate to output deviations is set to 0.10. The parameter governing the persistence of the interest rate rule ρ_R is set to 0.70. The volatility parameter σ_ξ is set to 0.3%.²³ The parameter that determines the steady-state value of inflation Π_{ss} is set to match the average level of inflation in the data.

I now turn to the calibration of the other three models for which the benchmark model is compared to. ENDO 2 is the same as ENDO 1 but with no policy uncertainty $\sigma_\xi = 0$. Thus in ENDO 2, there is only one exogenous shock. In the two exogenous growth models, the common parameters with the growth model are kept the same to facilitate a direct comparison. However, since the trend component is exogenous in those models, this will entail additional parameters governing the exogenous dynamics of the trend. In the the exogenous growth model with a stochastic trend EXO 2, the process for the low-frequency productivity growth shock is calibrated so that the expected productivity growth component matches key features of the endogenously expected growth component in EXO 1. In particular, the persistence parameter ρ_x is set to match the first autocorrelation of $E[\Delta z_t]$ with that of ENDO 1. The volatility parameter σ_x is set to match the volatility of $E[\Delta z_t]$ with that of ENDO 1. The parameter governing the average level of the trend μ is set at 0.55% to match the average level of output growth. EXO 1 is the same as EXO 2 except

²¹To provide further discipline on the calibration of ρ , note that since the ENDO models imply the TFP decomposition, $\Delta z_t = \Delta a_t + \Delta n_t$, we can project log TFP growth on log growth of the R&D stock to back out the residual Δa_t . The autocorrelations of the extracted residual $\Delta \hat{a}_t$ show that we cannot reject that it is white noise. Hence, in levels, it must be the case that a_t is a persistent process to be consistent with this empirical evidence. In our benchmark calibration, the annualized value of ρ is .94.

²²See, for example, Clarida, Galí, and Gertler (2000) and Rudebusch (2002).

²³See, for example, Smets and Wouters (2007).

that σ_x is set to 0 so that the trend is deterministic.

4.2 Macroeconomic Dynamics

Table 2 reports the key macroeconomic moments. Note that the benchmark growth model (ENDO 1) closely fits key business cycle moments. Worth noting, the model is able to match investment volatilities, which is a challenging feature in the data to explain jointly with high risk premia in standard production-based models with recursive preferences.²⁴ In particular, when IES is high enough, small amounts of capital adjustment costs discourage investment volatility. However, for the same reason, small nominal interest rate shocks that induce movements in the real rate due to nominal rigidities, lead to strong incentives to adjust labor and capital inputs. Indeed, comparing ENDO 1 with the benchmark model without interest rate shocks (ENDO 2), one can see that the volatility of log hours growth, physical investment growth, and R&D investment growth are substantially larger in ENDO 1. Thus, incorporating standard New Keynesian features, nominal rigidities and policy shocks, helps alleviate a long-standing problem in standard real business cycle (RBC) models with recursive preferences.

At business cycle frequencies, the benchmark growth model performs at least as well as the exogenous growth counterparts, EXO 1 and EXO 2. This can be seen by comparing ENDO 1 with the last two columns of Table 2. At low-frequencies, the benchmark growth model generates substantial endogenous long-run uncertainty in aggregate growth rates, which is reflected in the sizeable volatility of expected productivity growth. This is reported in Table 3. When firms receive a positive productivity shock, they increase the levels of inputs persistently, including R&D. As with standard business cycle models, increasing the level of inputs will lead to persistent cyclical movements around the trend. However, the persistent increase in the R&D input will also generate persistence in productivity growth, as discussed in Section 3. Naturally, aggregate growth rates, such as consumption, dividends, and output, will inherit these long-run productivity dynamics. Hence, the model generates both high- and low-frequency movements in quantities with a single productivity shock. Empirically, this mechanism suggests that measures related to innovation should have forecasting ability for aggregate growth rates. This is verified in tables 6 and 7, which

²⁴See, for example, Croce (2008) and Kaltenbrunner and Lochstoer (2008).

report results from projecting future consumption growth and productivity growth over various horizons on the growth rate of the R&D capital stock. In the data, the growth rate of the R&D stock predicts future consumption and productivity growth over horizons for up to 4 years with significant point estimates and R^2 's that are increasing with the horizon. Qualitatively, the model reflects these patterns reasonably well. This gives empirical support to the notion of innovation-driven low-frequency variation in growth rates. Furthermore, these low-frequency growth dynamics are crucial for asset prices, which are discussed in the section below.

In contrast, in the exogenous growth model with a deterministic trend (EXO 1), expected productivity growth is approximately constant, as shown in section 3. Figure 2 plots the dynamic response of expected growth rates to a one standard deviation shock to productivity for both the endogenous and exogenous growth models. Table 9 further emphasizes this point by reporting the low-frequency volatilities of consumption, output, investment, and labor hours growth for ENDO 1 and EXO 1. The low-frequency component is identified using a bandpass filter with a bandwidth of 32 to 200 quarters.²⁵ Thus, incorporating persistent expected growth shocks is needed to generate significant low-frequency growth dynamics in the exogenous growth paradigm.

4.3 Asset Prices

This section discusses the asset pricing implications of the model, which critically hinges on the low-frequency dynamics of consumption growth and inflation that was outlined in the previous section. Since it is assumed that the agent has Epstein-Zin utility with a preference for an early resolution of uncertainty, this implies that not only are innovations to realized consumption and dividend growth priced, but also innovations to expected consumption and dividend growth. Table 4 reports the first and second moments of the risk-free rate, equity returns, and nominal yields. Quantitatively, the benchmark growth model is broadly consistent with the financial moments from the data. Also, note that comparing ENDO 1 with ENDO 2 demonstrates that the policy shock has a negligible impact on asset prices in this framework, because, as documented above, these shocks do not have a significant effect on the intertemporal distribution of risk, and primarily effect short-run investment and labor fluctuations. This highlights the importance of productivity shocks

²⁵Specifically, I use the bandpass filter from Christiano and Fitzgerald (2003).

in driving low-frequency dynamics (through the endogenous growth mechanism), which in turn is reflected in asset prices. Remarkably, the core results and rich dynamics of the benchmark growth model are generated with a single shock.

I first begin with a discussion of the model implications for real risks. Notably, the benchmark growth model (ENDO 1) generates a low and smooth risk-free rate and a sizeable equity premium. The equity premium is 3.67%. The volatility of the equity premium is 5.62%, which is a little over one-third of the historical volatility of the market excess return. Since the model is productivity-based, this number can be thought of as the productivity-driven fraction of historical excess return volatility. On the other hand, it is well known that dividend-specific shocks explain a good portion of stock return volatility.²⁶ To understand these results, it is useful to compare the benchmark endogenous growth model (ENDO 1) with the exogenous growth model with a deterministic trend (EXO 1). First note that while the two models have similar business cycle statistics, in EXO 1 the equity premium is close to zero and the risk-free rate is counterfactually high. This stark contrast between the two models is due to the fact that these two paradigms generate very different low-frequency growth dynamics, as described above. In particular, the strong propagation mechanism of the endogenous growth model generates substantial long-run uncertainty in aggregate growth rates while the EXO 1 model does not. As households with recursive preferences are very averse to uncertainty about long-run growth prospects, this implies that households have a much higher precautionary savings motive in the endogenous growth setting. In equilibrium, this leads to lower real interest rates in ENDO 1 than in EXO 1. Moreover, ENDO 1 also generates a substantial equity premium, which is due to aggregate growth rates, including consumption and dividends, naturally inheriting the innovation-driven low-frequency dynamics of productivity growth. Thus, the dividend claim is very risky, which is reflected in the sizeable equity premium.

EXO 2 incorporates exogenous long-run uncertainty through productivity growth, where the calibration of this shock is set so that it replicates the volatility and persistence of the expected productivity growth component in ENDO 1, which is reported in Table 3. In essence, incorporating this shock is a reduced-form way of capturing long-run uncertainty in productivity growth.

²⁶In particular, Ai, Croce and Li (2010) report that empirically the productivity-driven fraction of return volatility is around 6%, which is consistent with this quantitative finding.

Evidently, incorporating this shock helps the exogenous growth paradigm generate a larger equity premium and lower risk-free rate than in EXO 1. However, even though the expected productivity dynamics share very similar properties, note that the equity premium in EXO 2 is a bit smaller than in ENDO 1 and the risk-free rate is larger than ENDO 1. This is because in the endogenous growth paradigm, short-run and long-run shocks are positively related and therefore reinforce each other. In particular, a good productivity shock leads to an increase in factor inputs, including R&D, which directly raises short-run output. Furthermore, an increase in R&D will boost long-run growth, therefore implying a tight link between short-run and long-run growth dynamics.

Now I turn to the model implications for nominal yields, which are also reported in Table 4. Note that the benchmark growth model (ENDO 1) closely matches both the means and volatilities of nominal yields for maturities of 4, 8, 12, 16, and 20 quarters in the data. Also, the average 20 quarter minus 1 quarter yield spread is a little over 1% as in the data. In contrast, the yield spread is close to zero in the exogenous growth model with a deterministic trend (EXO 1) and negative in the exogenous growth model with a stochastic trend (EXO 2). These results are intimately linked to the long-run co-movement between inflation and consumption growth. In particular, long bonds are riskier than short bonds if they have lower expected real payoffs (expected inflation is high) in states where marginal utility is high (expected consumption growth is low). Thus, in this setting where the agent has recursive preferences, expected inflation and expected consumption growth need to be negatively related for the models to produce an upward-sloping average nominal yield curve and sizeable term spread.²⁷

I first begin with a discussion of the equilibrium inflation and growth dynamics. Note that in the bottom row of Figure 1 the benchmark model (ENDO 1) replicates the negative low-frequency patterns in consumption growth and inflation from the data. As before, the low-frequency component is identified using a bandpass filter, where the bandwidth is from 32 to 200 quarters. Indeed, Table 10 shows that the ENDO 1 closely matches low-frequency correlations between macro growth rates and inflation. Furthermore, expected consumption growth and expected inflation are strongly negatively correlated. To understand the mechanics behind these endogenous long-run dynamics,

²⁷Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2009) highlight this point in endowment economy setups where inflation and consumption growth are exogenously specified and assumed to be negatively correlated. Furthermore, they find strong empirical evidence for this negative relationship.

it is instructive to look at the impulse response functions from Figure 3. First, I will focus on the IRFs for ENDO 1 which correspond to the solid lines. When a good productivity shock is realized, expected growth rates, including consumption growth, increase persistently as discussed above. Now, I will also show that expected inflation declines persistently in response to a good productivity shock. Recall from Section 3, inflation depends positively on the wage to capital ratio and labor hours worked, and negatively on the productivity shock and the R&D stock to physical capital stock ratio:

$$\begin{aligned}\tilde{\pi}_t &= \gamma_1 \tilde{m}c_t + \gamma_2 E_t[\tilde{\pi}_{t+1}] \\ \tilde{m}c_t &= \tilde{w}_t + \alpha \tilde{l}_t - (1 - \alpha)\tilde{a}_t - (1 - \alpha)\tilde{n}_t\end{aligned}$$

where $\gamma_1 = \frac{\nu-1}{\phi_R} > 0$, $\gamma_2 = \beta \Delta Y_{ss}^{1-\frac{1}{\psi}} > 0$, and lowercase tilde variables denote log deviations from the steady-state. As discussed above, $\tilde{w}_t$ and $\tilde{l}_t$ increase in response to an increase in $\tilde{a}_t$. In addition, because the firm has very high incentives to take advantage of the good shock to increase long-run growth prospects, the stock of R&D increases more relative to the increase in physical capital. Thus, $\tilde{n}_t$ also increases. Quantitatively, the increase in the terms $(1 - \alpha)\tilde{a}_t$ and $(1 - \alpha)\tilde{n}_t$ dominates the increase in $\tilde{w}_t$ and $\alpha \tilde{l}_t$, so that marginal costs, and therefore inflation declines. In sum, the productivity shock drives a strong negative relationship between expected consumption growth and expected inflation. Moreover, these dynamics are reflected in asset markets by an upward sloping yield curve and sizeable term spread.

Importantly, there is strong empirical support for these model-implied inflation-growth dynamics. Barsky and Sims (2009) and Kurmann and Otrok (2011) show in a VAR that a positive shock to expected productivity growth (“news shock”) leads to large and persistent decline in inflation. In the benchmark growth model, fluctuations to expected productivity growth are driven by R&D rates. Specifically, in the benchmark growth model, the expected productivity growth component is the growth rate of the stock of R&D. A persistent increase in R&D during good times lowers marginal costs and inflation persistently. Indeed, in the data innovation rates and inflation share a strong negative relationship, as predicted by the model. The top left plot of Figure 1 provides visual evidence of the negative long-run relationship between R&D and inflation. The benchmark

model exhibits similar patterns which can be seen in the top right plot of Figure 1. As before, the low-frequency component is identified using a bandpass filter, where the bandwidth is from 32 to 200 quarters. Table 10 corroborates the visual evidence by showing that the long-run correlations between inflation and macro growth rates, including the R&D stock and measured productivity, are strongly negative in the data and the benchmark model. Furthermore, the model predicts that measures related to innovation should forecast inflation rates with negative loadings on the innovation variable. This is verified in Table 8, which reports the results from projecting future inflation rates on the growth rate of the R&D stock for horizons of one to four years. In the data, the R^2 values are sizeable and the point estimates are negative and statistically significant. Qualitatively, the model reflects these features reasonably well.

The negative long-run relationship between expected growth and inflation rates is also crucial for reconciling the empirical observation that increases in the term spread predict higher future economic growth.²⁸ In the benchmark model, a positive productivity shock leads to a persistent increase in expected growth and a persistent decline in inflation. Given that the monetary authority is assumed to follow a Taylor rule and aggressively responds to inflation deviations, a drop in inflation leads the monetary authority to lower the short-term nominal rate. A temporary decline in the short rate implies that future short rates are expected to rise, which leads to a steeper slope for the yield curve.²⁹ In sum, the model predicts that a rise in the slope of the yield curve is associated with an increase in future growth rates. This is verified in Table 5, which reports consumption growth forecasts with the 20 quarter yield spread. In the data, the R^2 values are sizeable and the point estimates are positive and statistically significant. In particular, the regressions from ENDO 1 produces R^2 that are of similar magnitude as the ones from the data and positive point estimates.

To highlight the importance of the endogenous growth mechanism for explaining the term structure, it is useful to compare the benchmark model (ENDO 1) with the exogenous growth models EXO 1 (deterministic trend) and EXO 2 (stochastic trend). In EXO 1, the average nominal yield curve is upward sloping, however, the average slope is counterfactually small, as reported in Table 4. This shortcoming is inherently related to the inability of the model to generate a

²⁸See, for example, Ang, Piazzesi, and Wei (2006) for empirical evidence that find that term spread forecasts future growth.

²⁹Kurmann and Otrok (2011) provide empirical support for this mechanism.

sizeable equity premium. Namely, the model lacks a strong propagation mechanism that generates quantitatively sufficient long-run consumption uncertainty. Figures 2 and 3 highlight this point. Note that the dashed lines correspond to the impulse response functions for EXO 1 in response to a productivity shock and the solid lines correspond to ENDO 1. Expected inflation and expected consumption growth are negatively related, which drive the upward sloping yield curve. While expected productivity growth is close to constant in EXO 1, consumption smoothing will drive persistence in consumption growth. However, this channel is quantitatively small, which can be readily seen in the impulse response for expected consumption growth and comparing it with the response from ENDO 1.

Interestingly, incorporating exogenous long-run uncertainty by adding a persistent expected productivity growth shock, as in EXO 2, makes the slope of the nominal yield curve negative. This counterfactual implication implies that incorporating this shock to the exogenous growth framework leads to a positive relationship between expected consumption growth and expected inflation.³⁰ In particular, a positive growth shock has a large wealth effect that reduces the incentives to work. Thus, wages need to rise sharply and persistently to induce households to supply labor in order to maintain the equilibrium level of consumption and output. Quantitatively, the large and persistent rise in wages along with the eventual increase in labor hours dominates the increase in the trend-capital ratio. Thus, marginal costs and inflation increase precisely when expected growth rates increase. These dynamics are depicted in Figure 4. To contrast, in the endogenous growth framework labor decisions affect long-run growth rates and agents have higher incentives to supply labor in good times to boost expected growth prospects. Thus, the incentives to work in good times are higher (than in the exogenous growth framework), which dampens the wealth effect of persistently higher future growth and allows the benchmark model maintain a strong negative relationship between expected growth and inflation rates.

The positive relationship between expected consumption growth and inflation in EXO 2 also implies that a decline in the slope of the yield curve forecasts higher future economic growth, which is counterfactual. A positive long-run productivity growth shock increases expected consumption growth and inflation. An increase in inflation leads the monetary authority to increase in the

³⁰This is also documented in Rudebusch and Swanson (2012).

nominal short rate aggressively. A temporary increase in the short rate implies that future short rates are expected to fall. Consequently, the slope of the nominal yield curve decreases. This intuition is verified in Table 5, which reports consumption growth forecasts with the 20 quarter yield spread. In the data, the R^2 values are sizeable and the point estimates are positive and statistically significant. The forecasting regressions from the model correspond to population values. In particular, the regressions from EXO produce negative point estimates. In sum, the endogenous growth margin is critical for reconciling nominal bond data.

4.4 Policy Experiments

While monetary policy shocks have a negligible impact on long-run growth dynamics and asset prices, this section demonstrates that changing the policy parameters can have a large quantitative impact on the intertemporal distribution of risk in the benchmark growth model. Specifically, changing the policy parameters alters the transmission of the productivity shock. This section explores the effects of varying the intensity of inflation and output stabilization on asset prices for a reasonable range of parameter values. In the model, monetary policy is characterized by a short-term nominal interest rate rule that responds to current inflation and output deviations. Due to the presence of nominal rigidities, changes in the short rate affect the real rate which alters real decisions, including R&D. Thus, monetary policy can influence trend growth dynamics. Moreover, by varying the policy parameters, even such short-run stabilization policies can have substantial effects on the level and dynamics of long-run growth, which in turn have important implications for real and nominal risks.

Figure 5 illustrates the effects of varying the policy parameter ρ_y , where a larger value means a more aggressive stance on output stabilization. In particular, more aggressive output stabilization implies that the short rate, and thus the real rate, will increase more in response to an increase in output. Since R&D rates are procyclical, a larger rise in the real rate will dampen the increase in R&D more and lower the volatility of expected growth rates. With less uncertainty about long-run growth prospects, the equity risk premium declines. On the other hand, more aggressive output stabilization amplifies the volatility of expected inflation. Since expected inflation is countercyclical, a sharper rise in the real rate depresses expected inflation even further. Greater uncertainty about

expected inflation makes long bonds even riskier, which increases the average slope of the nominal yield curve.

Figure 6 illustrates the effects of varying the policy parameter ρ_π , where a larger value means a more aggressive stance on inflation stabilization. In particular, more aggressive inflation stabilization implies that the short rate, and thus the real rate, will increase more in response to an increase in inflation. Since inflation and R&D rates are negatively correlated, a larger rise in the real rate will further depress R&D and thus, amplify R&D rates. Furthermore, more volatile R&D rates imply that expected growth rates are more volatile. Higher uncertainty about long-term growth prospects therefore increases the equity risk premium. Additionally, more aggressive inflation stabilization will naturally smooth expected inflation which lowers the average slope of the nominal yield curve. Thus, inflation and output stabilization have opposite effects on asset markets. In short, these results suggest that monetary policy, even when targeting short-run deviations, can have a substantial impact on asset markets by distorting long-run growth and inflation dynamics.

5 Conclusion

This paper examines the nominal yield curve implied by a stochastic endogenous growth model with imperfect price adjustment. In good times when productivity is high, firms increase R&D, which raises expected growth rates. In addition, the increase in productivity and R&D lowers marginal costs persistently. As firms face downward sloping demand curves, a fall in marginal costs leads firms to lower prices, which in the aggregate, leads to a persistent decline in inflation. Thus, the model endogenously generates low-frequency movements in productivity growth and inflation that are negatively related. From the perspective of a bondholder, these dynamics imply that long bonds have lower expected payoffs than short bonds when long-run growth prospects are expected to be grim. When households have recursive preferences, this implies that long bonds have particularly low payoffs when marginal utility is high. As a result, the model generates an upward sloping average nominal yield curve and sizeable term spread. In addition, when the monetary authority follows a Taylor rule, the negative relationship between expected growth and inflation implies that a rise in the slope of the yield curve predicts higher future growth.

More broadly, this paper offers a unified framework to study macroeconomics and asset pricing. Notably, incorporating the endogenous growth margin with assumption of recursive preferences into a standard New Keynesian DSGE framework allows this class of models to explain a wide array of stylized facts in asset pricing. From a macroeconomic perspective, the endogenous growth mechanism allows these models to rationalize both high- and low-frequency dynamics of aggregate variables. From a production-based asset pricing perspective, incorporating sticky prices and nominal interest rate shocks allow these models to explain the observed high investment volatility.

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Appendix A. Data

Annual and quarterly data for consumption, capital investment, and GDP are from the Bureau of Economic Analysis (BEA). Annual data on private business R&D investment is from the survey conducted by the National Science Foundation (NSF). Annual data on the stock of private business R&D is from the Bureau of Labor Statistics (BLS). Annual productivity data is obtained from the BLS and is measured as multifactor productivity in the private nonfarm business sector. Quarterly total wages and salaries data are from the BEA. Quarterly hours worked data are from the BLS. The wage rate is defined as the total wages and salaries divided by hours worked. The sample period is for 1953-2008, since R&D data is only available during that time period. Consumption is measured as expenditures on nondurable goods and services. Capital investment is measured as private fixed investment. Output is measured as GDP. The variables are converted to real using the Consumer Price Index (CPI), which is obtained from the Center for Research in Security Prices (CRSP). The inflation rate is computed by taking the log return on the CPI index.

Monthly nominal return and yield data are from CRSP. The real market return is constructed by taking the nominal value-weighted return on the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) and deflating it using the CPI. The real risk-free rate is constructed by using the nominal average one-month yields on treasury bills and taking out expected inflation.³¹ Nominal yield data for maturities of 4, 8, 12, 16, and 20 quarters are from the CRSP Fama-Bliss discount bond file. The 1 quarter nominal yield is from the the CRSP Fama risk-free rate file.

Appendix B. Intermediate Goods Firm Problem

The Lagrangian for intermediate firm i 's problem is

$$\begin{aligned}
 V^{(i)}(\mathcal{P}_{i,t-1}, K_{i,t}, N_{i,t}; \mathbf{\Upsilon}_t) &= F(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t) - \frac{\mathcal{W}_{i,t}}{\mathcal{P}_t} L_{i,t} - I_{i,t} - S_{i,t} - G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}; \mathcal{P}_t, Y_t) \\
 &+ E_t \left[M_{t+1} V^{(i)}(\mathcal{P}_{i,t}, K_{i,t+1}, N_{i,t+1}; \mathbf{\Upsilon}_{t+1}) \right] \\
 &+ \Lambda_{i,t} \left\{ \frac{\mathcal{P}_{i,t}}{\mathcal{P}_t} - J(K_{i,t}, N_{i,t}, L_{i,t}; A_t, N_t, Y_t) \right\} \\
 &+ Q_{i,k,t} \left\{ (1 - \delta_k) K_{i,t} + \Phi_k \left(\frac{I_{i,t}}{K_{i,t}} \right) K_{i,t} - K_{i,t+1} \right\} \\
 &+ Q_{i,n,t} \left\{ (1 - \delta_n) N_{i,t} + \Phi_n \left(\frac{S_{i,t}}{N_{i,t}} \right) N_{i,t} - N_{i,t+1} \right\}
 \end{aligned}$$

³¹We model the monthly time series process for inflation using an AR(4).

The first-order conditions are

$$\begin{aligned}
0 &= -G_{i,1,t} + E_t[M_{t+1}V_{p,t+1}^{(i)}] + \frac{\Lambda_{i,t}}{\mathcal{P}_t} \\
0 &= -1 + Q_{i,k,t}\Phi'_{i,k,t} \\
0 &= -1 + Q_{i,n,t}\Phi'_{i,n,t} \\
0 &= E_t[M_{t+1}V_{k,t+1}^{(i)}] - Q_{i,k,t} \\
0 &= E_t[M_{t+1}V_{n,t+1}^{(i)}] - Q_{i,n,t} \\
0 &= F_{i,l,t} - \frac{\mathcal{W}_{i,t}}{\mathcal{P}_t} - \Lambda_{i,t}J_{i,l,t}
\end{aligned}$$

The envelope conditions are

$$\begin{aligned}
V_{p,t}^{(i)} &= -G_{i,2,t} \\
V_{k,t}^{(i)} &= F_{i,k,t} - \Lambda_{i,t}J_{i,k,t} + Q_{i,k,t} \left(1 - \delta_k - \frac{\Phi'_{i,k,t}I_{i,t}}{K_{i,t}} + \Phi_{i,k,t} \right) \\
V_{n,t}^{(i)} &= F_{i,n,t} - \Lambda_{i,t}J_{i,n,t} + Q_{i,n,t} \left(1 - \delta_n - \frac{\Phi'_{i,n,t}S_{i,t}}{N_{i,t}} + \Phi_{i,n,t} \right)
\end{aligned}$$

where $Q_{i,k,t}$, $Q_{i,n,t}$, and $\Lambda_{i,t}$ are the shadow values of physical capital, R&D capital and price of intermediate goods, respectively.³² Define the following terms from the equations above:

$$\begin{aligned}
G_{i,1,t} &= \phi_R \left(\frac{\mathcal{P}_{i,t}}{\Pi_{ss}\mathcal{P}_{i,t-1}} - 1 \right) \frac{Y_t}{\Pi_{ss}\mathcal{P}_{i,t-1}} \\
G_{i,2,t} &= -\phi_R \left(\frac{\mathcal{P}_{i,t}}{\Pi_{ss}\mathcal{P}_{i,t-1}} - 1 \right) \frac{Y_t\mathcal{P}_{i,t}}{\Pi_{ss}\mathcal{P}_{i,t-1}^2} \\
\Phi_{i,k,t} &= \frac{\alpha_{1,k}}{1 - \frac{1}{\zeta_k}} \left(\frac{I_{i,t}}{K_{i,t}} \right)^{1 - \frac{1}{\zeta_k}} + \alpha_{2,k} \\
\Phi_{i,n,t} &= \frac{\alpha_{1,n}}{1 - \frac{1}{\zeta_n}} \left(\frac{S_{i,t}}{N_{i,t}} \right)^{1 - \frac{1}{\zeta_n}} + \alpha_{2,n} \\
\Phi'_{i,k,t} &= \alpha_{1,k} \left(\frac{I_{i,t}}{K_{i,t}} \right)^{-\frac{1}{\zeta_k}} \\
\Phi'_{i,n,t} &= \alpha_{1,n} \left(\frac{S_{i,t}}{N_{i,t}} \right)^{-\frac{1}{\zeta_n}}
\end{aligned}$$

³² $\Phi_{i,k,t} \equiv \Phi_k \left(\frac{I_{i,t}}{K_{i,t}} \right)$, $\Phi_{i,n,t} \equiv \Phi_n \left(\frac{S_{i,t}}{N_{i,t}} \right)$, $\Phi'_{i,k,t} \equiv \alpha_{1,k} \left(\frac{I_{i,t}}{K_{i,t}} \right)^{-\frac{1}{\zeta_k}}$, $\Phi'_{i,n,t} \equiv \alpha_{1,n} \left(\frac{S_{i,t}}{N_{i,t}} \right)^{-\frac{1}{\zeta_n}}$ are defined for notational convenience.

$$\begin{aligned}
F_{i,k,t} &= \frac{\alpha \left(1 - \frac{1}{\nu}\right) Y_t^{\frac{1}{\nu}} X_{i,t}^{1-\frac{1}{\nu}}}{K_{i,t}} \\
F_{i,n,t} &= \frac{\eta(1-\alpha) \left(1 - \frac{1}{\nu}\right) Y_t^{\frac{1}{\nu}} X_{i,t}^{1-\frac{1}{\nu}}}{N_{i,t}} \\
F_{i,l,t} &= \frac{(1-\alpha) \left(1 - \frac{1}{\nu}\right) Y_t^{\frac{1}{\nu}} X_{i,t}^{1-\frac{1}{\nu}}}{L_{i,t}} \\
J_{i,k,t} &= \frac{-\left(\frac{\alpha}{\nu}\right) Y_t^{\frac{1}{\nu}} X_{i,t}^{-\frac{1}{\nu}}}{K_{i,t}} \\
J_{i,n,t} &= \frac{-\left(\frac{\eta(1-\alpha)}{\nu}\right) Y_t^{\frac{1}{\nu}} X_{i,t}^{-\frac{1}{\nu}}}{N_{i,t}} \\
J_{i,l,t} &= \frac{-\left(\frac{1-\alpha}{\nu}\right) Y_t^{\frac{1}{\nu}} X_{i,t}^{-\frac{1}{\nu}}}{L_{i,t}}
\end{aligned}$$

Substituting the envelope conditions and definitions above, the first-order conditions can be expressed as:

$$\begin{aligned}
\frac{\Lambda_{i,t}}{\mathcal{P}_t} &= \phi_R \left(\frac{\mathcal{P}_{i,t}}{\Pi_{ss} \mathcal{P}_{i,t-1}} - 1 \right) \frac{Y_t}{\Pi_{ss} \mathcal{P}_{i,t-1}} - E_t \left[M_{t+1} \phi_R \left(\frac{\mathcal{P}_{i,t+1}}{\Pi_{ss} \mathcal{P}_{i,t}} - 1 \right) \frac{Y_{t+1} \mathcal{P}_{i,t+1}}{\Pi_{ss} \mathcal{P}_{i,t}^2} \right] \\
Q_{i,k,t} &= \frac{1}{\Phi'_{i,k,t}} \\
Q_{i,n,t} &= \frac{1}{\Phi'_{i,n,t}} \\
\\
Q_{i,k,t} &= E_t \left[M_{t+1} \left\{ \frac{\alpha(1-\frac{1}{\nu}) Y_{t+1}^{\frac{1}{\nu}} X_{i,t+1}^{1-\frac{1}{\nu}}}{K_{i,t+1}} + \frac{\Lambda_{i,t+1} \left(\frac{\alpha}{\nu}\right) Y_{t+1}^{\frac{1}{\nu}} X_{i,t+1}^{-\frac{1}{\nu}}}{K_{i,t+1}} \right\} \right] \\
&\quad + E_t \left[M_{t+1} Q_{i,k,t+1} \left(1 - \delta_k - \frac{\Phi'_{i,k,t+1} I_{i,t+1}}{K_{i,t+1}} + \Phi_{i,k,t+1} \right) \right] \\
Q_{i,n,t} &= E_t \left[M_{t+1} \left\{ \frac{\eta(1-\alpha)(1-\frac{1}{\nu}) Y_{t+1}^{\frac{1}{\nu}} X_{i,t+1}^{1-\frac{1}{\nu}}}{N_{i,t+1}} + \frac{\Lambda_{i,t+1} \left(\frac{\eta(1-\alpha)}{\nu}\right) Y_{t+1}^{\frac{1}{\nu}} X_{i,t+1}^{-\frac{1}{\nu}}}{N_{i,t+1}} \right\} \right] \\
&\quad + E_t \left[M_{t+1} Q_{i,n,t+1} \left(1 - \delta_n - \frac{\Phi'_{i,n,t+1} S_{i,t+1}}{N_{i,t+1}} + \Phi_{i,n,t+1} \right) \right] \\
\frac{\mathcal{W}_{i,t}}{\mathcal{P}_t} &= \frac{(1-\alpha)(1-\frac{1}{\nu}) Y_t^{\frac{1}{\nu}} X_{i,t}^{1-\frac{1}{\nu}}}{L_{i,t}} + \Lambda_{i,t} \frac{\left(\frac{1-\alpha}{\nu}\right) Y_t^{\frac{1}{\nu}} X_{i,t}^{-\frac{1}{\nu}}}{L_{i,t}}
\end{aligned}$$

Appendix C. Derivation of the New Keynesian Phillips Curve

Define $MC_t \equiv \frac{W_t}{MPL_t}$ and $MPL_t \equiv (1 - \alpha) \frac{Y_t}{L_t}$ for real marginal costs and the marginal product of labor, respectively. Rewrite the price-setting equation of the firm in terms of real marginal costs

$$\nu MC_t - (\nu - 1) = \phi_R \left(\frac{\Pi_t}{\Pi_{ss}} - 1 \right) \frac{\Pi_t}{\Pi_{ss}} - E_t \left[M_{t+1} \phi_R \left(\frac{\Pi_{t+1}}{\Pi_{ss}} - 1 \right) \frac{\Delta Y_{t+1} \Pi_{t+1}}{\Pi_{ss}} \right]$$

Log-linearizing the equation above around the nonstochastic steady-state gives

$$\tilde{\pi}_t = \gamma_1 \tilde{mc}_t + \gamma_2 E_t[\tilde{\pi}_{t+1}]$$

where $\gamma_1 = \frac{\nu-1}{\phi_R}$, $\gamma_2 = \beta \Delta Y_{ss}^{1-\frac{1}{\psi}}$, and lowercase variables with a tilde denote log deviations from the steady-state.³³

Substituting in the expression for the marginal product of labor and imposing the symmetric equilibrium conditions, real marginal costs can be expressed as

$$MC_t = \frac{W_t L_t}{(1 - \alpha) K_t^\alpha (A_t N_t L_t)^{1-\alpha}}$$

Define the following stationary variables: $\bar{W}_t \equiv \frac{W_t}{K_t}$ and $\bar{N}_t \equiv \frac{N_t}{K_t}$. Thus, we can rewrite the expression above as

$$MC_t = \frac{\bar{W}_t L_t^\alpha}{(1 - \alpha) (A_t \bar{N}_t)^{1-\alpha}}$$

Log-linearizing this expression yields

$$\tilde{mc}_t = \tilde{w}_t + \alpha \tilde{l}_t - (1 - \alpha) \tilde{a}_t - (1 - \alpha) \tilde{n}_t$$

where lowercase variables with a tilde denote log deviations from the steady-state.

³³In a log-linear approximation, the relationship between the price adjustment cost parameter ϕ_R and the fraction of firms that resetting prices $(1 - \theta_c)$ from a Calvo pricing framework is given by: $\phi_R = \frac{(\nu-1)\theta_c}{(1-\theta_c)(1-\beta\theta_c)}$. Further, the average price duration implied by the Calvo pricing framework is $\frac{1}{1-\theta_c}$ quarters.

Table 1: Quarterly Calibration

		ENDO 1	ENDO 2	EXO 1	EXO 2
β	Subjective Discount Factor	0.9965	0.9965	0.9965	0.9965
ψ	Elasticity of Intertemporal Substitution	2.0	2.0	2.0	2.0
γ	Risk Aversion	10.0	10.0	10.0	10.0
α	Capital Share	0.33	0.33	0.33	0.33
ρ	Persistence of a_t	0.985	0.985	0.985	0.985
σ	Volatility of Productivity Shock ϵ	1.36%	1.36%	1.36%	1.36%
ϕ_R	Magnitude of Price Adjustment Costs	30.0	30.0	30.0	30.0
ν	Price Elasticity for Intermediate Good	6.0	6.0	6.0	6.0
η	Degree of Technological Appropriability	0.1	0.1	0.0	0.0
δ_k	Depreciation Rate of Capital Stock	0.02	0.02	0.02	0.02
δ_n	Depreciation Rate of R&D Stock	0.0375	0.0375	-	-
ζ_k	Capital Adjustment Cost Parameter	4.7	4.7	4.7	4.7
ζ_n	R&D Capital Adjustment Cost Parameter	3.3	3.3	-	-
ρ_r	Degree of Monetary Policy Inertia	0.7	0.7	0.7	0.7
ρ_π	Sensitivity of Interest Rate to Inflation	1.5	1.5	1.5	1.5
ρ_y	Sensitivity of Interest Rate to Output	0.10	0.10	0.10	0.10
σ_ξ	Volatility of ξ_t	0.3%	0.0%	0.3%	0.3%
μ	Trend Growth Rate	-	-	0.55%	0.55%
ρ_x	Persistence of x	-	-	0.971	0.971
σ_x	Volatility of ϵ_x	-	-	0.00%	0.063%

This table reports the quarterly calibration for the benchmark endogenous growth model (ENDO 1), endogenous growth model without monetary policy uncertainty (ENDO 2), exogenous model with a deterministic trend (EXO 1), and exogenous growth model with a stochastic trend (EXO 2).

Table 2: Macroeconomic Moments

	Data	ENDO 1	ENDO 2	EXO 1	EXO 2
1st Moments					
$E[\Delta y]$	2.20%	2.20%	2.20%	2.20%	2.20%
$E[\Delta \pi]$	3.74%	3.74%	3.74%	3.74%	3.74%
2nd Moments					
$\sigma_{\Delta c}/\sigma_{\Delta y}$	0.64	0.60	0.72	0.54	0.64
$\sigma_{\Delta l}/\sigma_{\Delta y}$	0.92	0.98	0.19	0.76	1.10
$\sigma_{\Delta i}/\sigma_{\Delta c}$	4.38	4.38	2.61	4.96	3.84
$\sigma_{\Delta s}/\sigma_{\Delta c}$	3.44	3.44	2.43	-	-
$\sigma_{\Delta c}$	1.42%	1.42%	1.40%	1.24%	1.42%
σ_{π}	1.64%	1.73%	1.60%	1.98%	3.01%
σ_w	2.04%	2.85%	2.59%	5.05%	5.46%
$AC1(\Delta c)$	0.21	0.26	0.28	0.04	0.13
$AC1(\pi)$	0.89	0.89	0.95	0.94	0.95

This table presents first and second macroeconomic moments from the benchmark endogenous growth model (ENDO 1), endogenous growth model without monetary policy uncertainty (ENDO 2), exogenous growth model with a deterministic trend (EXO 1), exogenous growth model with a stochastic trend (EXO 2), and the data. The models are calibrated at a quarterly frequency and the the reported means and volatilities are annualized. Macro data are obtained from the BEA, BLS, and NSF. The data sample is 1953-2008.

Table 3: Expected Productivity Growth Dynamics

	ENDO 1	ENDO 2	EXO 2
$AC1(E[\Delta Z])$	0.90	0.90	0.90
$\sigma(E[\Delta Z])$	0.58%	0.56%	0.58%

This table reports the annualized persistence and standard deviation of the expected growth rate component of productivity growth from the benchmark endogenous growth model (ENDO 1), the benchmark endogenous growth model without policy uncertainty (ENDO 2), and the exogenous growth model with a stochastic trend (EXO 2). The exogenous productivity dynamics from EXO 2 are calibrated to match the endogenous productivity dynamics from ENDO 2.

Table 4: Asset Pricing Moments

	Data	ENDO 1	ENDO 2	EXO 1	EXO 2
1st Moments					
$E[r_d^* - r_f]$	5.84%	3.67%	3.64%	0.05%	1.84%
$E[r_f]$	1.62%	1.01%	1.02%	2.57%	2.06%
$E[y^{(4)}]$	5.29%	5.15%	5.25%	6.23%	5.67%
$E[y^{(8)}]$	5.48%	5.43%	5.52%	6.24%	5.61%
$E[y^{(12)}]$	5.66%	5.69%	5.78%	6.24%	5.50%
$E[y^{(16)}]$	5.80%	5.94%	6.03%	6.24%	5.37%
$E[y^{(20)}]$	5.89%	6.18%	6.27%	6.25%	5.10%
$E[y^{(20)} - y^{(1)}]$	1.01%	1.20%	1.19%	0.02%	-0.57%
2nd Moments					
$\sigma(r_d - r_f)$	17.87%	5.62%	5.35%	1.57%	3.99%
$\sigma(r_f)$	0.67%	0.68%	0.38%	0.37%	0.68%
$\sigma(y^{(4)})$	2.99%	2.38%	2.33%	3.88%	5.94%
$\sigma(y^{(8)})$	2.96%	2.29%	2.26%	3.80%	5.83%
$\sigma(y^{(12)})$	2.88%	2.21%	2.18%	3.72%	5.71%
$\sigma(y^{(16)})$	2.83%	2.14%	2.11%	3.64%	5.58%
$\sigma(y^{(20)})$	2.77%	2.07%	2.04%	3.56%	5.46%
$\sigma(y^{(20)} - y^{(1)})$	1.00%	0.74%	0.36%	0.70%	0.93%

This table presents annual first and second asset pricing moments from the benchmark endogenous growth model (ENDO 1), endogenous growth model without monetary policy uncertainty (ENDO 2), exogenous growth model with a deterministic trend (EXO 1), exogenous growth model with a stochastic trend (EXO 2), and the data. The models are calibrated at a quarterly frequency and the moments are annualized. Since the equity risk premium from the models is unlevered, we follow Boldrin, Christiano, and Fisher (2001) and compute the levered risk premium from the model as: $r_{d,t+1}^* - r_{f,t} = (1 + \kappa)(r_{d,t+1} - r_{f,t})$, where r_d is the unlevered return and κ is the average aggregate debt-to-equity ratio, which is set to $\frac{2}{3}$. Monthly return and price data are from CRSP and the corresponding sample moments are annualized. The data sample is 1953-2008.

Table 5: Consumption Growth Forecasts with 20Q Yield Spread

Horizon (k)	Data			ENDO 1		EXO 2	
	β	S.E.	R^2	β	R^2	β	R^2
1Q	0.762	0.188	0.095	0.699	0.034	-0.620	0.040
4Q	2.421	0.671	0.146	2.950	0.082	-1.856	0.065
8Q	2.866	1.263	0.080	8.635	0.102	-3.286	0.078

This table presents quarterly consumption growth forecasting regressions from the data and from the benchmark endogenous growth model (ENDO 1) and the exogenous model with a stochastic trend (EXO 2) for horizons (k) of 1, 4, and 8 quarters. Specifically, log real consumption growth is projected on the 20 quarter nominal yield spread, $\Delta c_{t,t+1} + \dots + \Delta c_{t+k-1,t+k} = \alpha + \beta(y_t^{(20)} - y^{(1)}) + \nu_{t,t+k}$. In the data, the regression is estimated via OLS with Newey-West standard errors with $k-1$ lags. The model regression results correspond to the population values. Overlapping quarterly observations are used. Consumption data are from the BEA and nominal yield data are from CRSP.

Table 6: Productivity Growth Forecasts with R&D Growth

Horizon (k)	Data			ENDO 1	
	β	S.E.	R^2	β	R^2
1	0.431	0.190	0.113	2.946	0.176
2	0.820	0.315	0.192	5.606	0.243
3	1.230	0.452	0.262	8.193	0.283
4	1.707	0.522	0.376	10.588	0.302

This table presents annual productivity growth forecasting regressions from the data and from the benchmark endogenous growth model (ENDO 1) for horizons (k) of one year to four years. Specifically, log productivity growth is projected on log R&D stock growth, $\Delta z_{t,t+1} + \dots + \Delta z_{t+k-1,t+k} = \alpha + \beta \Delta n_t + \nu_{t,t+k}$. In the data the regression is estimated via OLS with Newey-West standard errors with $k-1$ lags. The model regression results correspond to the population values. Overlapping annual observations are used. Multifactor productivity and R&D stock data is from the BLS.

Table 7: Consumption Growth Forecasts with R&D Growth

Horizon (k)	Data			ENDO 1		
	β	S.E.	R^2	β	S.E.	R^2
1	0.217	0.084	0.094	1.993	0.678	0.177
2	0.395	0.178	0.115	3.643	1.207	0.214
3	0.540	0.276	0.132	5.044	1.766	0.219
4	0.703	0.347	0.168	6.221	2.333	0.215

This table presents annual consumption growth forecasting regressions from the data and from the benchmark endogenous growth model (ENDO 1) for horizons (k) of one year to four years. Specifically, real consumption growth is projected on log R&D stock growth, $\Delta c_{t,t+1} + \dots + \Delta c_{t+k-1,t+k} = \alpha + \beta \Delta n_t + \nu_{t,t+k}$. Both in the data and in the model, the regression is estimated via OLS with Newey-West standard errors with $k - 1$ lags. The model estimates correspond to 200 simulations of 56 years. Overlapping annual observations are used. Consumption data is from the BEA and R&D stock data is from the BLS.

Table 8: Inflation Forecasts with R&D Growth

Horizon (k)	Data			ENDO 1		
	β	S.E.	R^2	β	S.E.	R^2
1	-0.543	0.168	0.156	-6.414	0.650	0.613
2	-1.065	0.409	0.171	-11.757	1.454	0.582
3	-1.560	0.671	0.177	-16.132	2.407	0.530
4	-2.015	0.927	0.177	-19.674	3.466	0.478

This table presents annual inflation growth forecasting regressions from the data and from the benchmark endogenous growth model (EGR 1) for horizons (k) of one year to four years. Specifically, log inflation is projected on log R&D stock growth, $\pi_{t,t+1} + \dots + \pi_{t+k-1,t+k} = \alpha + \beta \Delta n_t + \nu_{t,t+k}$. Both in the data and in the model, the regression is estimated via OLS with Newey-West standard errors with $k - 1$ lags. The model estimates correspond to 200 simulations of 56 years. Overlapping annual observations are used. Consumption data is from the BEA and R&D stock data is from the BLS.

Table 9: Volatility of Low-Frequency Components

	Data	ENDO 1	EXO 1
$\sigma(\Delta c)$	0.75%	0.86%	0.65%
$\sigma(\Delta y)$	1.16%	1.19%	0.99%
$\sigma(\Delta i)$	3.06%	2.35%	2.09%
$\sigma(\Delta l)$	1.29%	0.36%	0.32%

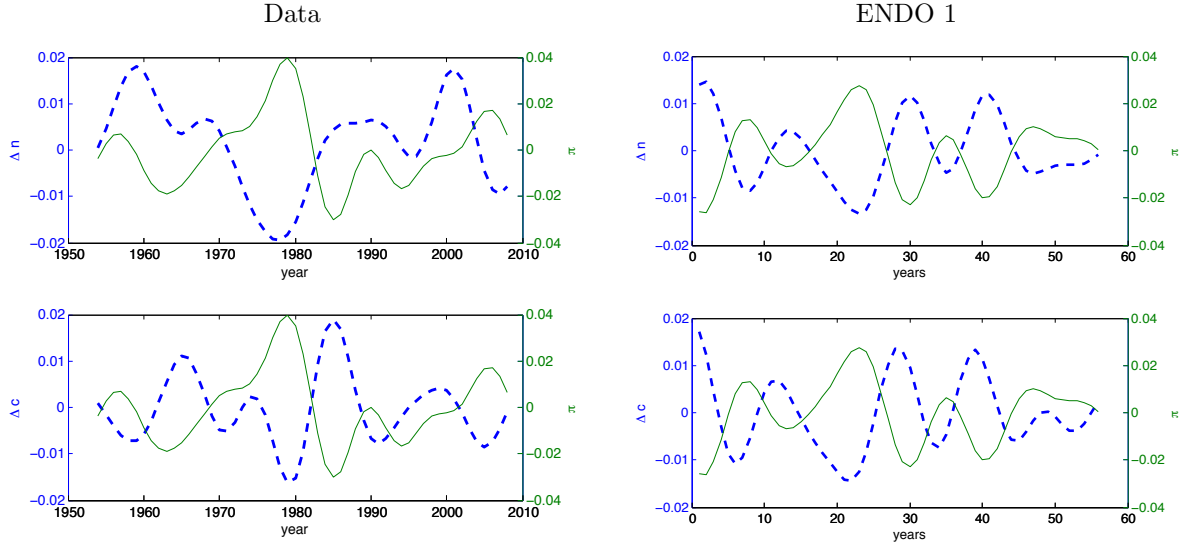
This table reports the annualized volatilities of low-frequency components of log consumption, output, investment, and labor growth rates from the data and from the benchmark endogenous growth model (ENDO 1) and the neoclassical model with a deterministic trend (EXO 1). The bandpass filter from Christiano and Fitzgerald (2003) is used to isolate the low frequency component. The low-frequency component is defined as a bandwidth of 32 to 200 quarters. Quarterly macro data are from the BEA and BLS. Inflation data are from CRSP.

Table 10: Low-Frequency Correlations

	Data	ENDO 1	EXO 2
$corr(\Delta c, \pi)$	-0.77	-0.84	-0.10
$corr(\Delta y, \pi)$	-0.74	-0.71	-0.01
$corr(\Delta z, \pi)$	-0.52	-0.64	0.08
$corr(\Delta n, \pi)$	-0.63	-0.90	-
$corr(E[\Delta c], E[\pi])$	-	-0.97	0.49

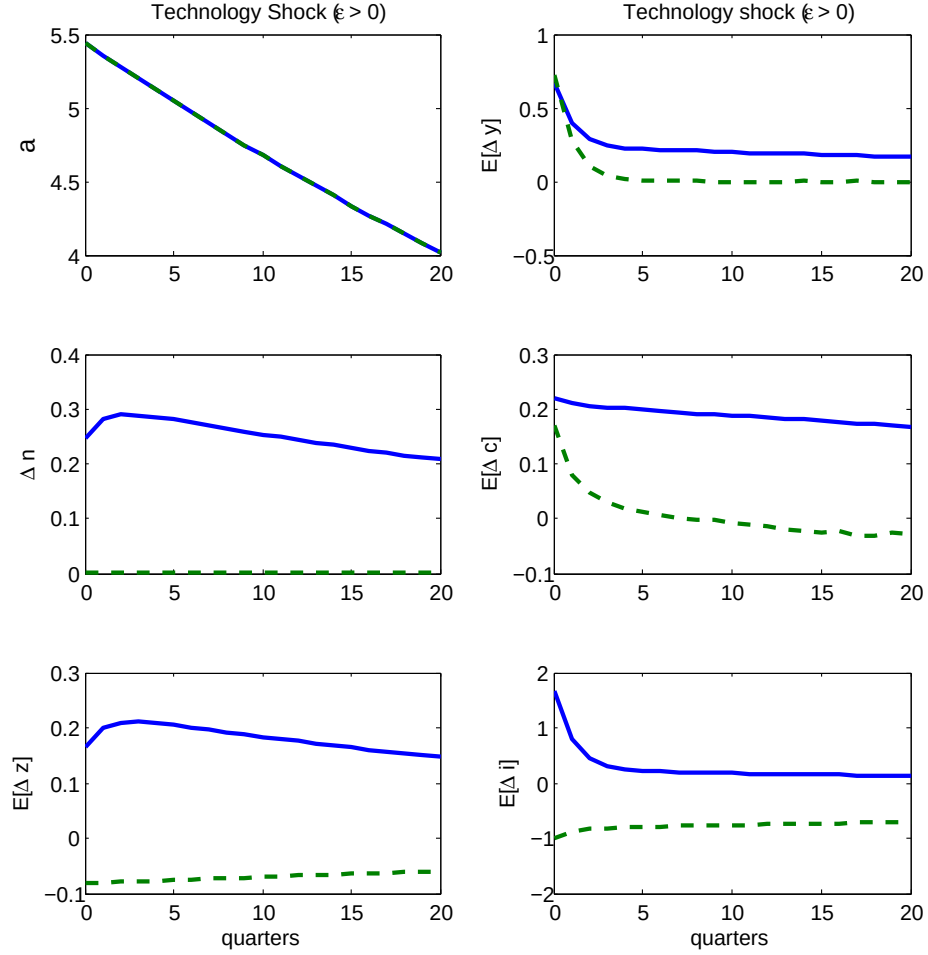
This table reports the correlation of low-frequency components of log consumption, output, productivity, and R&D stock growth with inflation from the data and from the benchmark endogenous growth model (ENDO 1) and the exogenous growth model with a stochastic trend (EXO 2). The bandpass filter from Christiano and Fitzgerald (2003) is used to isolate the low-frequency component. The low-frequency component is defined as a bandwidth of 32 to 200 quarters. Quarterly output data is from the BEA and BLS. Inflation data are from CRSP.

Figure 1: Long-Run Co-Movement



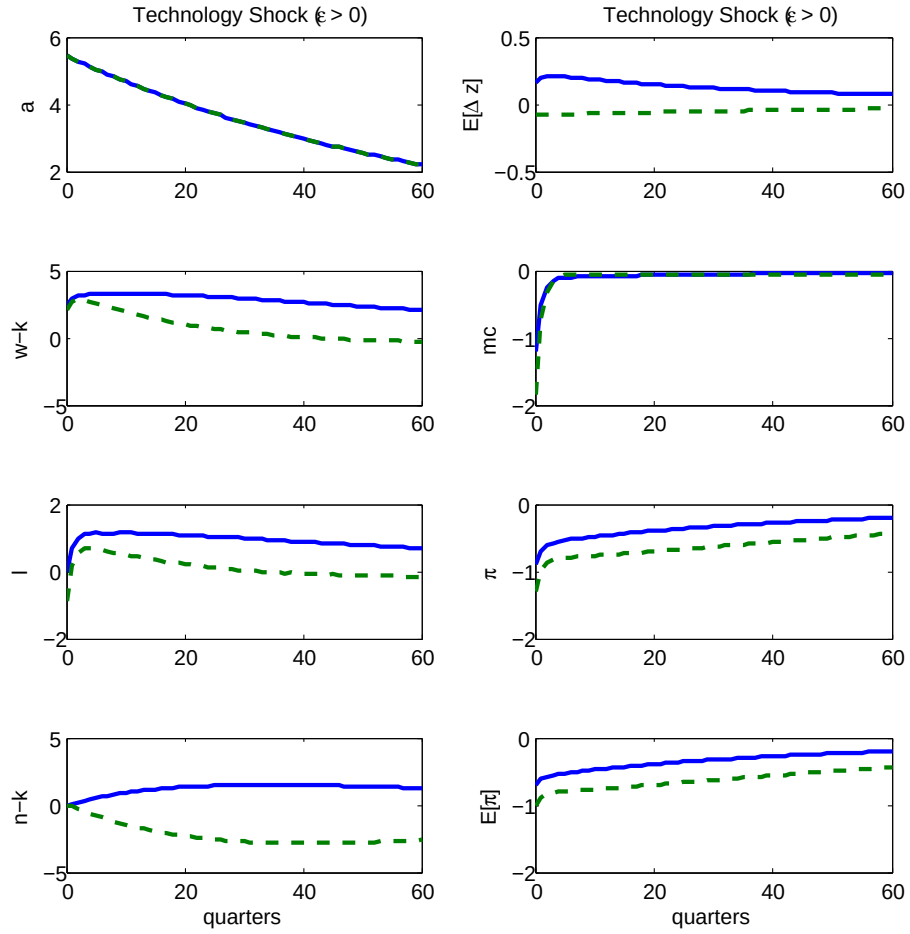
The top left panel plots the low-frequency components of both log consumption growth (dashed line, scale on left axis) and log inflation (solid line, scale on right axis) from the data. The bottom left panel plots the low-frequency components of both log R&D stock growth (dashed line, scale on left axis) and log inflation (solid line, scale on right axis) from the data. The top right panel plots the low-frequency components of both log consumption growth (dashed line, scale on left axis) and log inflation (solid line, scale on right axis) from the benchmark model. The bottom right panel plots the low-frequency components of both log R&D stock growth (dashed line, scale on left axis) and log inflation (solid line, scale on right axis) from the benchmark model. The low-frequency component is obtained using the bandpass filter from Christiano and Fitzgerald (2003) and selecting a bandwidth of 32 to 200 quarters. Macro data are from the BEA and BLS. Inflation data are from CRSP.

Figure 2: Expected Growth Rates



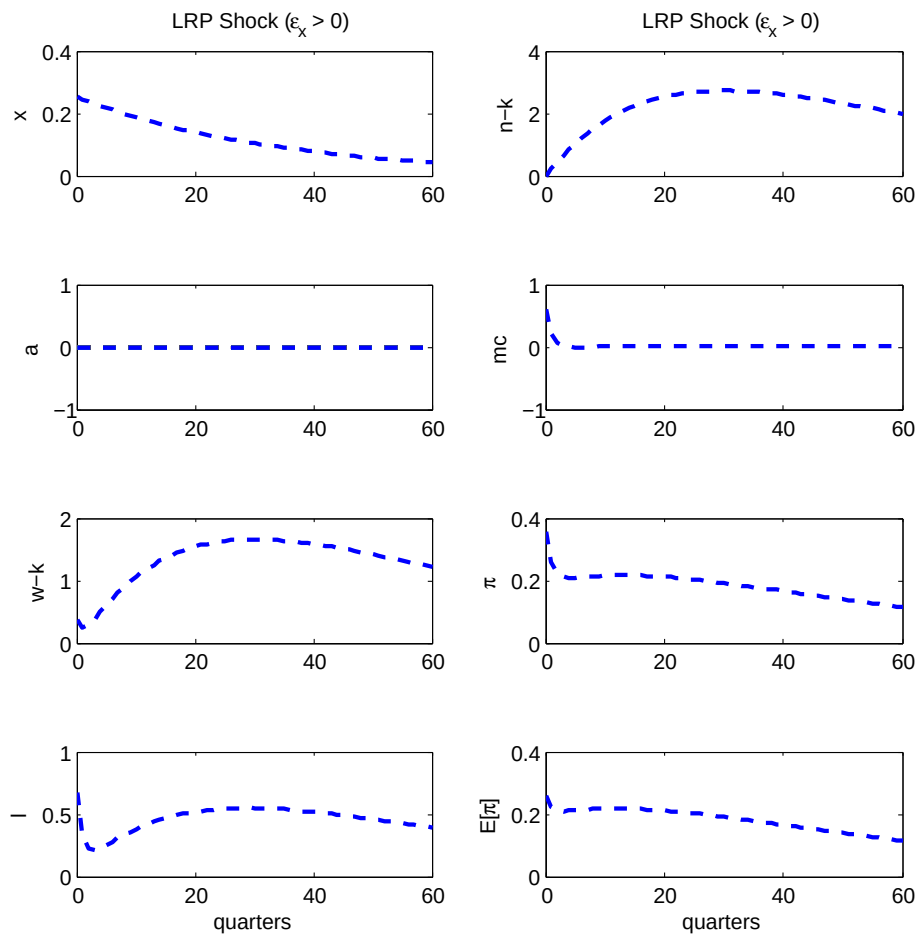
This figure shows quarterly log-deviations from the steady state for the benchmark endogenous growth model ENDO 1 (solid line) and the exogenous growth model with a deterministic trend NCL 1 (dashed line) from a one standard deviation shock to productivity. All deviations are in annualized percentage units.

Figure 3: Expected Inflation and Growth Mechanisms (Productivity Shock)



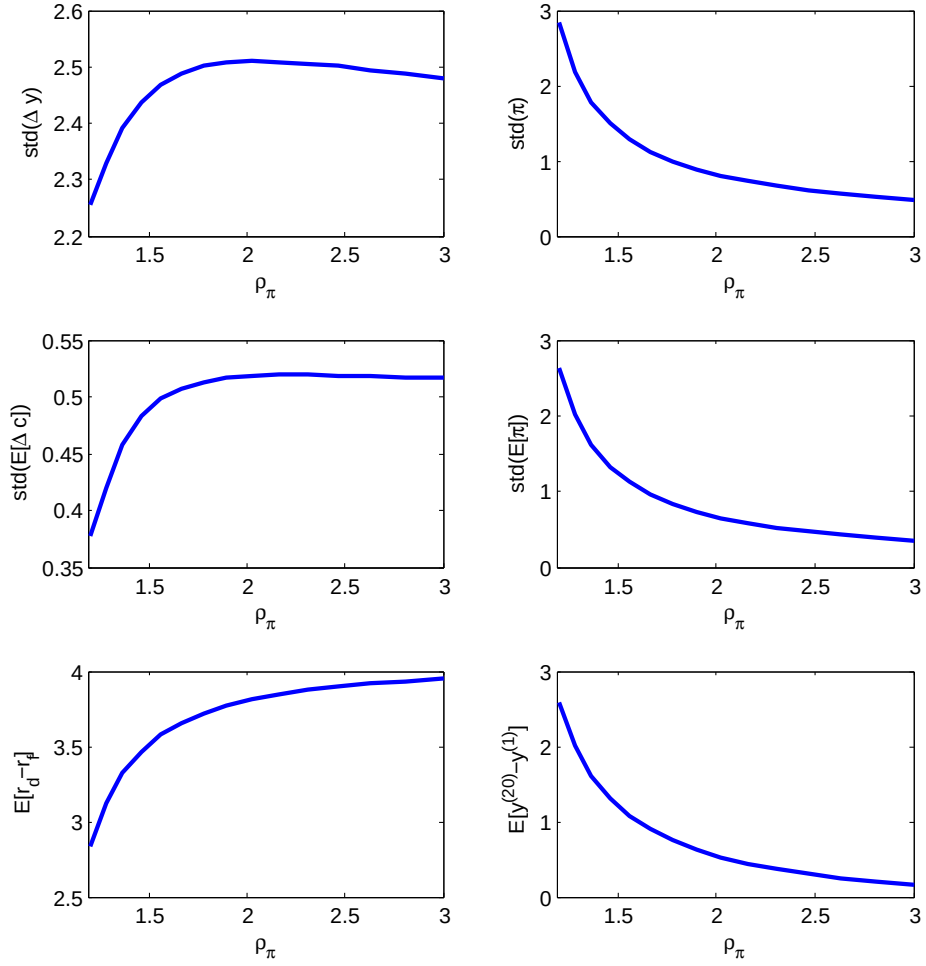
This figure shows quarterly log-deviations from the steady state for the benchmark endogenous growth model ENDO 1 (solid line) and the neoclassical models (dashed line) from a one standard deviation shock to technology. All deviations are in annualized percentage units.

Figure 4: Expected Inflation and Growth Mechanisms (Expected Growth Shock)



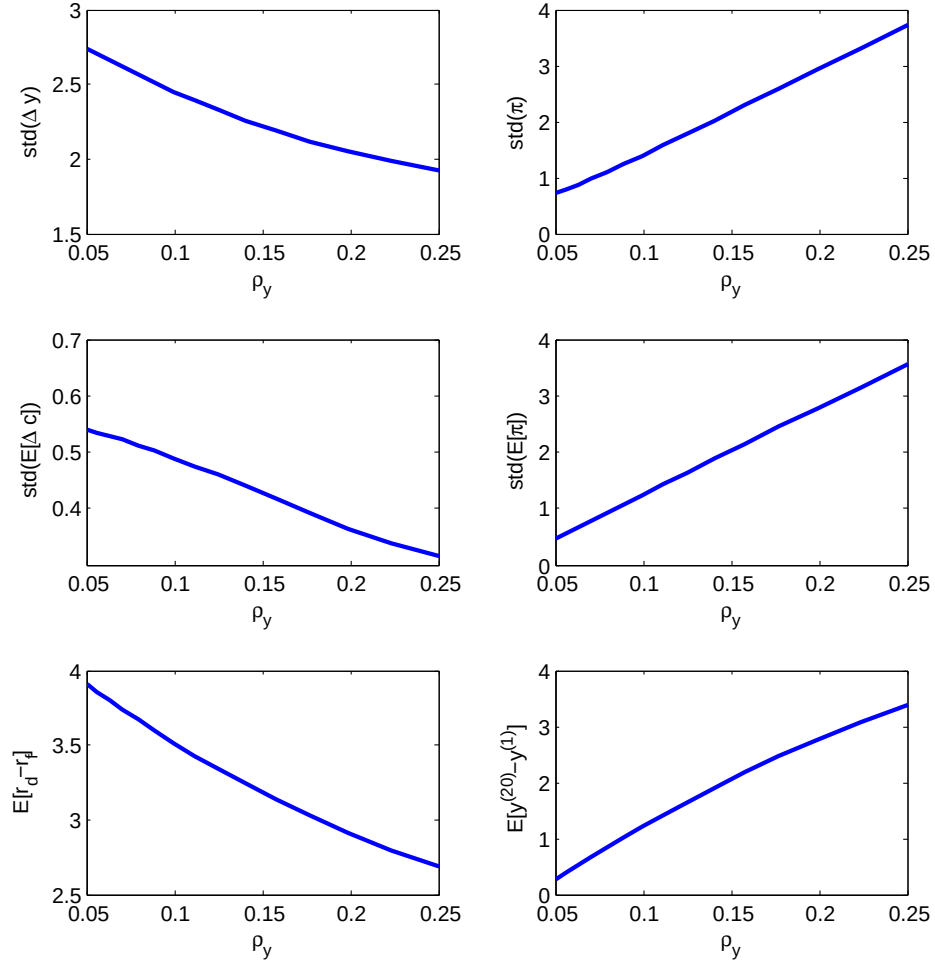
This figure shows quarterly log-deviations from the steady state for the exogenous growth model model with a stochastic trend EXO 2 from a one standard deviation shock to expected productivity growth. All deviations are in annualized percentage units.

Figure 5: Varying Intensity of Inflation Stabilization



This figure plots the impact of varying the policy parameter ρ_π on output growth volatility, inflation volatility, volatility of expected consumption growth, volatility of expected inflation, equity premium, and average nominal yield spread in the benchmark growth model. Values on y-axis are in annualized percentage units.

Figure 6: Varying Intensity of Output Stabilization



This figure plots the impact of varying the policy parameter ρ_y on output growth volatility, inflation volatility, volatility of expected consumption growth, volatility of expected inflation, equity premium, and average nominal yield spread in the benchmark growth model. Values on y-axis are in annualized percentage units.