Innovation, Growth and Asset Prices

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Abstract

Asset prices reflect anticipations of future growth. We examine the asset pricing implications of a production economy whose long-term growth prospects are endogenously determined by innovation and R&D. In equilibrium, R&D endogenously drives a small, persistent component in productivity which generates long-run uncertainty about economic growth. With recursive preferences, households fear that persistent slowdowns in economic growth are accompanied by low asset valuations and command high risk premia in asset markets. Empirically, we find substantial evidence for innovation-driven low-frequency movements in aggregate growth rates and asset market valuations. In short, equilibrium growth is risky.

Keywords: Endogenous growth, asset pricing, innovation, R&D, productivity, low-frequency cycles, business cycle propagation, recursive preferences.

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1 Introduction

Asset prices reflect anticipations of future growth. Likewise, long-term growth prospects mirror an economy's innovative potential. At the aggregate level, such innovation is reflected in the sustained growth of productivity. Empirical measures of innovation, such as R&D expenditures, are typically quite volatile and fairly persistent. Such movements in the driving forces of growth prospects should naturally be reflected in the dynamics of growth rates themselves. Indeed, in US post-war data productivity growth has undergone long and persistent swings.[1] Similarly, innovation-driven growth waves associated with the arrival of new technologies such as television, computers, the internet, to name a few, are well documented.[2] Asset prices reflect this low-frequency variation in growth prospects. In particular, if agents fear that a persistent slowdown in economic growth will lower asset prices, such movements will give rise to high risk premia in asset markets.

In this paper, we use a tractable model of innovation and R&D in order to link asset prices and aggregate risk premia to endogenous movements in long-term growth prospects. More specifically, our setup has two distinguishing features. First, we embed a stochastic model of endogenous growth based on industrial innovation[3] into an otherwise standard real business cycle model. Here technological progress and sustained growth is determined endogenously by the creation of new patents and technologies through R&D. New patents facilitate the production of a final consumption good and can be thought of as intangible capital. Second, we assume that households have recursive preferences, so that they care about uncertainty regarding long-term growth prospects.

Our results suggest that extending macroeconomic models to account for the endogeneity of innovation and long-term growth goes some way towards an environment that jointly captures the dynamics of quantities and asset markets. When calibrated to match the empirical evidence on productivity and long-run economic growth, our model can quantitatively rationalize key features of asset returns in the data. In particular, it generates a realistic equity premium and a low and stable risk-free interest rate without relying on excessively high risk aversion. Moreover, it generates a sizeable spread between the returns on physical capital and intangible capital, which is

[1] See e.g. Gordon (2010), and Jermann and Quadrini (2007)

[2] See e.g. Helpman (1998), and Jovanovic and Rousseau (2003)

[3] Following Romer (1990) and Grossman and Helpman (1991)

commonly related to the value premium in the data. In short, equilibrium growth is risky.

Our model supports the notion that movements in long-term growth prospects are a significant source of risk priced in asset markets. Such 'long-run risks' (in the sense of Bansal and Yaron (2004)) arise endogenously in our production economy suggesting that stochastic models of endogenous growth are a useful framework for general equilibrium asset pricing. At the center of this framework is a strong propagation and amplification mechanism for shocks which is tightly linked to the joint dynamics of innovation and asset prices. High equilibrium returns provide strong incentives for agents to engage in innovation and investing in R&D. This pricing effect reinforces the impact of exogenous shocks, thus providing an amplification mechanism. On the other hand, R&D leads to the development of new technologies which will persistently boost aggregate growth, so that aggregate growth appears in long waves, thus providing a propagation mechanism. Such endogenous persistence feeds back into asset prices with recursive preferences. When prolonged slumps in economic growth coincide with low asset valuations, households will require high risk premia in asset markets.

To the extent that asset prices are informative about welfare and the cost of fluctuations (as in Alvarez and Jermann, 2004), understanding the links between risk, innovation and growth is important for policy design. By distorting the intertemporal distribution of consumption, policies that affect firms' innovation decisions will affect the amount and dynamics of risk in the economy, and hence welfare and risk premia. While the real business business cycle model by construction is silent about this dimension of policy design, our model provides a framework to examine these linkages further. Croce, Nguyen and Schmid (2011, 2012) take some steps in this direction in the context of fiscal policy and Kung (2011) in the context of monetary policy.

Formally, we first show that in the model innovation and R&D endogenously drive a small, but persistent component in the growth rate of measured aggregate productivity. More specifically, we decompose productivity growth into a high-frequency component driven by an exogenous shock, as well as an endogenous component driven by R&D. While the shock induces fluctuations at business cycle frequency comparable to standard macroeconomic models, the innovation process in the model translates this disturbance into an additional, slow-moving component generating macroeconomic movements at lower frequencies. Naturally, these productivity dynamics induce persistent

uncertainty about the economy's long-term growth prospects that will be reflected in the dynamics of aggregate quantities.

Our model thus allows to identify economic sources of long-run risks in the data. In particular, it identifies R&D and innovation as economic sources of a predictable component in productivity, sometimes referred to as long-run productivity risk (as in Croce (2008), Gomes, Kogan and Yogo (2009), Backus, Routledge and Zin (2007, 2010), Favilukis and Lin (2010, 2011)). Indeed, in line with the predictions of the model, we provide novel empirical evidence that measures of innovation have significant predictive power for aggregate growth rates including productivity, consumption, output and cash flows at longer horizons.

Persistent variation in consumption and cash flow growth is reflected in risk premia in asset markets given our preference specification. With recursive Epstein-Zin utility with a preference for early resolution of uncertainty not only are innovations to realized consumption and dividend growth priced, but also innovations to expected consumption and dividend growth. The propagation mechanism in the model translates shocks into innovations to expected consumption growth, generating endogenous long-run risks in consumption, and innovations to expected dividend growth, generating realistic low-frequency movements in price-dividend ratios. Furthermore, in the model, physical capital is endogenously more exposed to predictable variation in growth than intangible capital, which generates a sizeable value spread.

Our paper is related to a number of different strands of literature in asset pricing, economic growth and macroeconomics. The economic mechanisms driving the asset pricing implications are closely related to Bansal and Yaron (2004). In a consumption-based model, Bansal and Yaron directly specify both consumption and dividend growth to contain a small, persistent component. This specification along with the assumption of Epstein-Zin recursive utility with a preference for early resolution of uncertainty, allows them to generate high equity premia as compensation for these long-run risks. While the empirical evidence for the long-run channel is still somewhat controversial, the ensuing literature on long-run risk quantitatively explains a wide range of patterns in asset markets, such as those in equity, government, corporate bond, foreign exchange and derivatives markets. We contribute to this literature by showing that predictable movements in growth prospects are an equilibrium outcome of stochastic models of endogenous growth and by providing

novel empirical evidence identifying economic sources of long-run risks in the data.

A number of recent papers have examined the link between technological growth and asset prices. Garleanu, Panageas and Yu (2011) model technological progress as the arrival of large, infrequent technological innovations and show that the endogenous adoption of these innovations leads to predictable movements in consumption growth and expected excess returns. Garleanu, Kogan and Panageas (2011) examine the implications of the arrival of new technologies for existing firms and their workers, and show that in an overlapping-generations model innovation creates a systematic risk factor labeled displacement risk. Pastor and Veronesi (2009) explain bubble-like behavior of stock markets in the 1990s by the arrival of new technologies.

While our model has implications for consumption dynamics and asset returns that are related to these models, our approach is quite different and complementary. In these models of technology adoption, the arrival of new technologies is assumed to be exogenous, while we examine the asset pricing and growth dynamics implications of the endogenous creation of new technologies by means of R&D, which leads to a distinct set of empirical predictions. Moreover, by embedding a model of endogenous technological progress into a real business cycle model, our paper provides a straightforward and tractable extension of the workhorse model of modern macroeconomics.

In this respect, the paper is closer to recent attempts to address asset pricing puzzles within versions of the canonical real business cycle model. Starting from Jermann (1998) and Boldrin, Christiano, Fisher (2001), recent examples include Campanale, Castro and Clementi (2008), Kaltenbrunner and Lochstoer (2008), Ai (2008) and Kuehn (2008), who explore endogenous long-run consumption risks in real business cycle models with recursive preferences, and Gourio (2009, 2010) who examines disaster risks. Particularly closely related are recent papers by Croce (2008), Backus, Routledge, Zin (2007, 2010), Gomes, Kogan, Yogo (2009) and Favilukis and Lin (2010, 2011) who examine the implications of long-run productivity risk with recursive preferences for the equity premium, and the cross-section of stock returns, respectively. While they specify long-run productivity risk exogenously, our model shows how such risk arises endogenously and can be linked to innovation. Tallarini (2000) considers the separate effects of risk and risk aversion on quantities with recursive preferences, while we investigate how risk and risk premia affect growth. Much like us, Eberly and Wang (2009, 2010) also examine a multi-sector model of endogenous growth with recursive

preferences, but operate with an AK-framework and focus on the effects of capital reallocation on growth. Our cross-sectional return implications are related to Lin (2009), Gala (2010) and Kogan and Papanikolaou (2010) who examine the effects of technological progress on the cross-section of returns.

Methodologically, our paper builds on and is closely related to recent work by Comin and Gertler (2007) and Comin, Gertler and Santacreu (2009). Building on the seminal work by Romer (1990) and Grossman and Helpman (1991), these authors integrate innovation and adoption of new technologies into a real business cycle model and show that the resulting stochastic endogenous growth model features rich movements at a lower-than-business-cycle-frequency, which they label medium term business cycles. We contribute to this literature by linking medium term cycles to long run risks and aggregate risk premia, and examining its asset pricing implications with recursive preferences. Moreover, while they consider low-frequency movements are around a trend, we focus on the low frequency movements of the trend growth rate. This is an important distinction from an asset pricing perspective.

More generally, our paper also contributes to the literature linking the endogenous growth literature and the business cycle literatures (Jones, Manuelli and Stacchetti (2000)), and the literature on firm dynamics over the business cycle (see e.g. Bilbiie, Ghironi and Melitz (2012) and Clementi and Palazzo (2010) for recent examples).

The paper is structured as follows. In section 2 we describe our benchmark model. In section 3 we qualitatively explore the growth and productivity processes arising in equilibrium and detail their links with the real business cycle model. We examine its quantitative implications for productivity, macroeconomic quantities and asset prices in section 4, along with a number of empirical predictions. Section 5 concludes.

2 Model

We start by describing out benchmark endogenous growth model. We embed a model of industrial innovation in the tradition of Romer (1990) into a fairly standard macroeconomic model with convex adjustment costs and recursive Epstein-Zin preferences. In the model, rather than assuming

exogenous technological progress, growth instead arises through research and development (R&D) investment. R&D investment leads to the creation of intermediate goods or new patents used in the production of a final consumption good. An increasing number of intermediate goods is the ultimate source of sustained growth, hence the model is a version of an expanding-variety model of endogenous growth.

Household The representative household has Epstein-Zin preferences defined over consumption:

$$U_t = \left\{ (1-\beta) C_t^{\frac{1-\gamma}{\theta}} + \beta (E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \tag{1}$$

where γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution, and $\theta \equiv \frac{1-\gamma}{1-1/\psi}$. When $\psi \neq \frac{1}{\gamma}$, the agent cares about news regarding long-run growth prospects. We will assume that $\psi > \frac{1}{\gamma}$ so that the agent has a preference for early resolution of uncertainty and dislikes shocks to long-run expected growth rates.

The household maximizes utility by participating in financial markets and by supplying labor. Specifically, the household can take positions $\mathcal{Z}_t$ in the stock market, which pays an aggregate dividend $\mathcal{D}_t$, and in the bond market, B_t. Accordingly, the budget constraint of the household becomes

$$C_t + \mathcal{Q}_t \mathcal{Z}_{t+1} + B_{t+1} = W_t L_t + (\mathcal{Q}_t + \mathcal{D}_t)\mathcal{Z}_t + R_t B_t \tag{2}$$

where $\mathcal{Q}_t$ is the stock price, R_t is the gross risk free rate, W_t is the wage and L_t denotes hours worked.

As described above, the production side of the economy consists of several sectors, so that the aggregate dividend can be further decomposed into the individual payouts of these sectors, in a way to be described below.

As usual, the setup implies that the stochastic discount factor in the economy is given by

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \frac{[\mathbb{E}_t(U_{t+1}^{1-\gamma})]^{\frac{\gamma-1/\psi}{1-\gamma}}}{U_{t+1}^{\gamma-1/\psi}} \quad . \tag{3}$$

where the second term, involving continuation utilities, captures preferences concerning uncertainty about long-run growth prospects. Furthermore, since the agent has no disutility for labor, she will supply her entire endowment, which we normalized to unity.

Final Goods Sector There is a representative firm that uses capital K_t, labor L_t and a composite of intermediate goods G_t to produce the final (consumption) good according to the production technology

$$Y_t = (K_t^{\alpha}(\Omega_t L_t)^{1-\alpha})^{1-\xi} G_t^{\xi} \tag{4}$$

where the composite G_t is defined as

$$G_t \equiv \left[\int_0^{N_t} X_{i,t}^{\frac{1}{\nu}}\, di\right]^{\nu}. \tag{5}$$

$X_{i,t}$ is intermediate good $i \in [0, N_t]$, where N_t is the measure of intermediate goods in use at date t, and α is the capital share, ξ is the intermediate goods share, and ν is the elasticity of of substitution between the intermediate goods. Note that $\nu > 1$ is assumed so that increasing the variety of intermediate goods raises the level of productivity in the final goods sector. This property is crucial for sustained growth. In our quantitative work, we will think of intermediate goods as new patents or intangible capital.

The productivity shock Ω_t is assumed to follow a stationary Markov process. Because of the stationarity of the forcing process, sustained growth will arise endogenously from the development of new intermediate goods. We will describe the R&D policy below.

The firm's objective is to maximize shareholder value. Taking the stochastic discount factor M_t as given, this can be formally stated as

$$\max_{\{I_t, L_t, K_{t+1}, X_{i,t}\}_{t\geq 0, i\in[0,N_t]}} E_0\left[\sum_{t=0}^{\infty} M_t D_t\right] \tag{6}$$

The firm's dividends are

$$D_t = Y_t - I_t - W_t L_t - \int_0^{N_t} P_{i,t} X_{i,t}\, di \tag{7}$$

where I_t is capital investment, W_t is the wage rate, and $P_{i,t}$ is the price per unit of intermediate good i, which the final goods firm takes as given. The last term captures the costs of buying intermediate goods at time t.

In line with the literature on production-based asset pricing, we assume that investment is subject to capital adjustment costs, so that the capital stock evolves as

$$K_{t+1} = (1-\delta)K_t + \Lambda\left(\frac{I_t}{K_t}\right)K_t. \tag{8}$$

Here, δ is the depreciation rate of capital and $\Lambda(\cdot)$ the capital adjustment cost function. $\Lambda(\cdot)$ is specified as in Jermann (1998)

$$\Lambda\left(\frac{I_t}{K_t}\right) \equiv \frac{\alpha_1}{1-\frac{1}{\zeta}}\left(\frac{I_t}{K_t}\right)^{1-\frac{1}{\zeta}} + \alpha_2$$

The parameter ζ represents the elasticity of the investment rate. The parameters α_1 and α_2 are set so that there are no adjustment costs in the deterministic steady state.

Denoting by q_t the shadow value of capital, the firm's optimality conditions are

$$\begin{aligned}
q_t &= \frac{1}{\Lambda_t'} \\
W_t &= (1-\alpha)(1-\xi)\frac{Y_t}{L_t} \\
1 &= E_t\left[M_{t+1}\left\{\frac{1}{q_t}\left(\alpha(1-\xi)\frac{Y_{t+1}}{K_{t+1}} + q_{t+1}(1-\delta) - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}\Lambda_{t+1}\right)\right\}\right] \\
P_{i,t} &= (K_t^{\alpha}(\Omega_t L_t)^{1-\alpha})^{1-\xi}\nu\xi\left[\int_0^{N_t} X_{i,t}^{\frac{1}{\nu}}\,di\right]^{\nu\xi-1}\frac{1}{\nu}X_{i,t}^{\frac{1}{\nu}-1}
\end{aligned}$$

where $\Lambda_t \equiv \Lambda\left(\frac{I_t}{K_t}\right)$ and $\Lambda_t' \equiv \Lambda'\left(\frac{I_t}{K_t}\right)$. The last equation determines the final good producer's demand for intermediate input. Importantly, that demand is procyclical, as it depends positively on Ω_t.

Intermediate Goods Sector Intermediate goods producers have monopoly power. Given the demand schedules set by the final good firm, monopolists producing the intermediate goods set the prices in order to maximize their profits. Intermediate goods producers transform one unit of the

final good in one unit of their respective intermediate good. In this sense production is "round-about" in that monopolists take final good production as given as they are tiny themselves. This fixes the marginal cost of producing one intermediate good at unity.

Focusing on symmetric equilibria, the monopolistically competitive characterization of the intermediate goods sector a la Dixit and Stiglitz (1977) implies

$$P_{i,t} = P_t = \nu \tag{9}$$

That is, each intermediate goods producer charges a markup $\nu > 1$ over marginal cost. Hence, intermediate profits are

$$\Pi_{i,t} = \Pi_t = (\nu - 1)X_t \tag{10}$$

where $X_{i,t} = X_t = \left(\frac{\xi}{\nu}\left(K_t^{\alpha}(\Omega_t L_t)^{1-\alpha}\right)^{1-\xi} N_t^{\nu\xi-1}\right)^{\frac{1}{1-\xi}}$. Consequently, the intermediate good input and hence monopoly profits are procyclical. The value of owning exclusive rights to produce intermediate good i is equal to the present discounted value of the current and future monopoly profits

$$V_{i,t} = V_t = \Pi_t + \phi E_t[M_{t+1}V_{t+1}] \tag{11}$$

where $1 - \phi$ is the probability that an intermediate good becomes obsolete. Again, given the procyclicality of profits, values of patents are procyclical as well. Since the values of patents are the payoffs to innovation, as described below, this implies that the returns to innovation are procyclical and risky.

R&D Sector Innovators develop new patents for intermediate goods used in the production of final output. They do so by conducting research and development, using the final good as input at unit cost. Patents of newly developed products can be sold to intermediate goods producers. Assuming that this market is competitive, the price of a new patent will equal its value to the new intermediate goods producer. For simplicity, we assume that households can directly invest in

research and development.

We link the evolution of the measure of intermediate goods or patents N_t to innovation as

$$N_{t+1} = \vartheta_t S_t + \phi N_t \tag{12}$$

where S_t denotes R&D expenditures (in terms of the final good) and ϑ_t represents the productivity of the R&D sector that is taken as exogenous by the R&D sector. In a similar spirit as Comin and Gertler (2006), we assume that this technology coefficient involves a congestion externality effect capturing decreasing returns to scale in the innovation sector

$$\vartheta_t = \frac{\chi \cdot N_t}{S_t^{1-\eta} N_t^{\eta}} \tag{13}$$

where $\chi > 0$ is a scale parameter and $\eta \in [0, 1]$ is the elasticity of new intermediate goods with respect to R&D. Since there is free entry into the R&D sector, the following break-even condition must hold:

$$E_t[M_{t+1} V_{t+1}](N_{t+1} - \phi N_t) = S_t \tag{14}$$

which says that the expected sales revenues equals costs, or equivalently, at the margin, $\frac{1}{\vartheta_t} = E_t[M_{t+1} V_{t+1}]$.

Resource Constraint Final output is used for consumption, investment in physical capital, factor input used in the production of intermediate goods, and R&D:

$$\begin{aligned} Y_t &= C_t + I_t + N_t X_t + S_t & (15) \\ &= C_t + I_t + N_t^{1-\nu} G_t + S_t & (16) \end{aligned}$$

where the last equality exploits the optimality conditions and the term $N_t^{1-\nu} G_t$ captures the costs of intermediate goods production. Given that $\nu > 1$ reflecting monopolistic competition, it follows that increasing product variety increases the efficiency of intermediate goods production, as the costs fall as N_t grows.

Stock Market We assume that the stock market value includes all the production sectors, namely the final good sector, the intermediate goods sector, as well as the research and development sector. The aggregate dividend then becomes

$$\mathcal{D}_t = D_t + \Pi_t N_t - S_t \tag{17}$$

Defining the stock market value to be the discounted sum of future aggregate dividends, exploiting the optimality conditions, this value can be rewritten as

$$\mathcal{Q}_t = q_t K_{t+1} + N_t(V_t - \Pi_t) + E_t \left[\sum_{i=0} M_{t+1+i} V_{t+i+1} (N_{t+i+1} - \phi N_{t+i}) \right] \tag{18}$$

as in Comin, Gertler, Santacreu (2009). The stock return is defined accordingly. Therefore, the stock market value comprises the current market value of the installed capital stock, reflected in the first term, the market value of currently used intermediate goods interpreted as patents or blueprints, reflected in the second term, as well as the market value of intermediate goods to be developed in the future, as reflected in the third term. Therefore, in addition to the tangible capital stock, the stock market values intangible capital as well as the option value of future intangibles.

Forcing Process We introduce uncertainty into the model by means of an exogenous shock Ω_t to the level of technology. We assume that $\Omega_t = e^{a_t}$, and $a_t = \rho a_{t-1} + \epsilon_t$, with $\epsilon_t \sim N(0, \sigma^2)$ and $\rho < 1$. Note first that this process is strictly stationary, so that sustained growth in the model will not arise through exogenous trend growth in exogenous productivity, but endogenously. Second, while formally, Ω_t resembles labor augmenting technology, it does not represent measured TFP in our setting. Rather, measured TFP in the model can be decomposed in an exogenous component, driven by Ω_t, and an endogenous component which is driven by the accumulation of intermediate goods and hence innovation, which is also the source of sustained growth. We discuss the dynamics of productivity in detail in section 3.

3 Equilibrium Growth and Productivity

In our benchmark model, sustained growth is an equilibrium phenomenon resulting from agents' decisions. Moreover, these decisions generate growth rate and productivity dynamics contrasting with those implied by more standard macroeconomic frameworks. In this section we describe these patterns qualitatively, while we will provide supportive empirical evidence and a quantitative analysis in the next section.

First, it is convenient to represent the aggregate production function in our benchmark model in a form that permits straightforward comparison with specifications used commonly in macroeconomic models where growth is given exogenously. To that end, note that using the equilibrium conditions derived above, final output can be rewritten as follows:

$$Y_t = \left(\frac{\xi}{\nu}\right)^{\frac{\xi}{1-\xi}} K_t^{\alpha} (\Omega_t L_t)^{1-\alpha} N_t^{\frac{\nu\xi-\xi}{1-\xi}} \tag{19}$$

For sustained growth to obtain in this setting we need to impose a parametric restriction. Technically, to ensure balanced growth, we need the aggregate production function to be homogeneous of degree one in the accumulating factors K_t and N_t. We will thus impose the parameter restriction that $\alpha + \frac{\nu\xi-\xi}{1-\xi} = 1$. In this case, we have a production function that resembles the standard neoclassical one with labor augmenting technology $Y_t = K_t^{\alpha}(Z_t L_t)^{1-\alpha}$ where total factor productivity (TFP) is

$$Z_t \equiv \overline{A}\Omega_t N_t \tag{20}$$

and $\overline{A} \equiv \left(\frac{\xi}{\nu}\right)^{\frac{\xi}{(1-\xi)(1-\alpha)}} > 0$ is a constant. The equilibrium productivity process thus contains a component driven by the exogenous forcing process, Ω_t, and an endogenous trend component reflecting the accumulation of intermediate goods, N_t.

In our quantitative work, we will contrast the implications of the benchmark with those of a nested standard real business cycle model with exogenous growth. We can achieve this by specifying the aggregate stock of R&D exogenously. More specifically, in the latter model, we specify TFP as $\tilde{Z}_t = \overline{A}\Omega_t \tilde{N}_t$ and a deterministic trend $\tilde{N}_t = e^{\mu t}$.

Hence, the fundamental difference between our model and the canonical real business cycle framework is that the trend component of the TFP process, N_t, is endogenous and fluctuates in our setup but exogenous and deterministic in the RBC model. Our benchmark model thus endogenously generates a stochastic trend, which is consistent with the evidence for OECD countries in Cogley (1990).

This stochastic trend is naturally reflected in the dynamics of productivity growth rates. Clearly, given a realistically persistent process for a_t, we have

$$\begin{aligned} \Delta z_t &= \Delta n_t + \Delta a_t \quad (21) \\ &\approx \Delta n_t + \epsilon_t \quad (22) \end{aligned}$$

where lowercase letters denote logs. In contrast, with a deterministic trend, we have $\Delta \widetilde{z}_t \approx \mu + \epsilon_t$. Accordingly, while in the counterpart with exogenous growth, productivity growth will be roughly i.i.d. and it will inherit a second component in the benchmark model which depends on the accumulation of patents. Therefore, qualitatively and quantitatively, the dynamics of productivity growth reflect the dynamics of innovation.

To see this more explicitly, rewrite the growth rate of productivity, ΔZ_t, as $\Delta Z_t = \Delta N_t \cdot \Delta \Omega_t$. Given a realistically persistent calibration of $\{\Omega_t\}$ in logs, we have $\Delta \Omega_t \approx e^{\epsilon_t}$. On the other hand, given the accumulation of N_t as $N_t = \vartheta_t \cdot S_{t-1} + \phi N_{t-1}$, the growth rate of patents becomes $\Delta N_t = \vartheta_t \cdot \widehat{S}_{t-1} + \phi$, where we set

$$\widehat{S}_t \equiv \frac{S_t}{N_t}$$

We will refer to $\widehat{S}_t$ as R&D *intensity*. Accordingly, we find $\Delta Z_t \approx (\vartheta_t \cdot \widehat{S}_{t-1} + \phi)(e^{\cdot \epsilon_t})$. Thus,

$$\begin{aligned} E_t[\Delta Z_{t+1}] &\approx E_t\left[(\vartheta_t \cdot \widehat{S}_t + \phi)(e^{\cdot \epsilon_{t+1}})\right] \quad (23) \\ &\approx \vartheta_t \cdot \widehat{S}_t + \phi \quad (24) \end{aligned}$$

Our model thus exhibits variation in expected growth driven by R&D intensity. Empirically, R&D intensity is a fairly persistent and volatile process. In a realistic calibration of the model, we therefore expect productivity growth to exhibit substantial low-frequency variation and persistent

uncertainty about growth prospects. Favorable economic conditions, as captured by a positive shock to a_t, also affect productivity and growth through their equilibrium effect on innovation and hence N_t, thus propagating shocks further. This is quite in contrast to the counterpart with exogenous growth, where expected productivity growth is approximately constant.

The equilibrium productivity growth dynamics implied by the model resemble closely those specified by Croce (2008). Croce specifies productivity to contain an i.i.d. component as well as a small, but persistent component. He refers to that latter component as long-run productivity risk and shows that this specification generates substantial risk premia in a production economy. While he exogenously specifies these dynamics, we show that such long-run productivity risk arises naturally in a setting with endogenous growth and that it is linked to innovation. Our model thus allows to empirically identify economic sources of long-run risk.

We can get further insights into the determinants of the stochastic trend by exploiting the specification of the innovation sector. From the law of motion for patents and the optimality condition for R&D it follows that the growth rate of the measure of intermediate goods satisfies

$$\frac{N_{t+1}}{N_t} = \phi + E_t \left[\chi M_{t+1} V_{t+1}\right]^{\frac{\eta}{1-\eta}} \tag{25}$$

$$= \phi + E_t \left[\chi^{\frac{1}{\eta}} \sum_{j=1}^{\infty} M_{t+j|t} \phi^{j-1} \Pi_{t+j}\right]^{\frac{\eta}{1-\eta}} \tag{26}$$

where $M_{t+j|t} \equiv \prod_s^j M_{t+s|t}$ is the j-step ahead stochastic discount factor and $M_{t|t} \equiv 1$. This implies that growth is directly related to the discounted value of future profits in the intermediate goods sector. This observation has two important implications. First, growth rates will naturally inherit the procyclicality of profits. Second, the average growth rate is endogenously related to the discount rate. Quantity dynamics therefore reflect risk premia. With recursive preferences, equilibrium growth will also depend on the endogenous amount of persistent long-term uncertainty. This is quite in contrast to the real business cycle model, where, as illustrated by Tallarini (2000), risk premia do not affect quantity dynamics.

4 Quantitative Implications

In this section we calibrate our model and explore its ability to replicate key moments of both macroeconomic quantities and asset returns. Rather than matching standard business cycle moments, we calibrate our model of endogenous growth to be quantitatively consistent with long-run dynamics of the aggregate economy, by which we mean isolating appropriate frequency bands in growth rates using a bandpass filter. On the other hand, we find it instructive to compare our benchmark model with a version in which trend growth is given exogenously. In the following, we refer to the benchmark endogenous growth model as ENDO, and the exogenous growth counterpart as EXO [4]. The models are calibrated at a quarterly frequency.

4.1 Parameter Choices

Our benchmark model has three main components: Recursive preferences, a technology to produce final consumption goods, and an innovation technology. Recursive preferences have been used extensively in recent work in asset pricing.[5] We follow this literature and set preference parameters to standard values that are also supported empirically.[6] The parameters related to the final goods sector are set to match long-run dynamics in the aggregate economy. We identify the long-run components of growth rates with movements at frequencies between 100 and 200 quarters, that we isolate using a bandpass filter. We follow Comin and Gertler (2006) in calibrating the parameters related to the intermediate goods and R&D sectors. These choices are also consistent with empirical evidence in the growth literature. Critically, satisfying balanced growth helps provide further restrictions on parameter values. Table 1 summarizes our parameter choices.

We begin with a description of the calibration of the preference parameters. The elasticity of intertemporal substitution ψ is set to value of 1.85[7]and the coefficient of relative risk aversion γ is

[4]While there is no exactly corresponding model with exogenous growth, we find our choice natural and it facilitates comparison. The main conclusions are robust across a broad spectrum of exogenous growth models. Extensive robustness checks with other exogenous growth specifications are available in a separate appendix on request.

[5]See Bansal and Yaron (2004).

[6]See Bansal, Yaron, and Kiku (2007) uses Euler conditions and a GMM estimator to provide empirical support for the parameter values.

[7]This choice is also consistent with the estimation evidence in Fernandez-Villaverde, Koijen, Rubio-Ramirez and van Binsbergen (2011).

set to a value of 10, which are standard values in the long-run risks literature. An intertemporal elasticity of substitution larger than one is consistent with the notion that an increase in uncertainty lowers the price-dividend ratio. Note that in this parametrization, $\psi > \frac{1}{\gamma}$, which implies that the agent dislikes shocks to expected growth rates and is particularly important for generating a sizeable risk premium in this setting. The subjective discount factor β is set to an annualized value of 0.984 so as to be consistent with the level of the riskfree rate.

In the final goods sector, the capital share α is set to .35, the intermediate goods (materials) share ξ is set to 0.5, and the depreciation rate of capital δ is set to 0.02. These three standard parameters are calibrated to match steady-state evidence. The capital adjustment cost function is standard in the production-based asset pricing literature.[8] The adjustment cost parameter ζ is set at 0.70 to match the relative volatility of long run consumption growth to output growth.

We now turn to the calibration of the parameters relating to the stationary productivity shock $a_t \equiv \log(\Omega_t)$. Note that this shock is different than the Solow residual since the final goods production technology includes a composite input consisting of an expanding variety of intermediate goods, as detailed in the previous section. The persistence parameter ρ is set to an annualized value of 0.95 and is calibrated to match the first autocorrelation of R&D intensity. Furthermore, this value for ρ allows us to be consistent with the first autocorrelations of the key quantity growth rates and productivity growth.[9] The volatility parameter σ is set at 1.75% to match long-run output growth volatility.

For the remaining parameters, the markup in the intermediate goods sector ν is set to a value of 1.65 and the elasticity of new intermediate goods with respect to R&D η is set to a value of 0.85. While the markup of intermediate inputs is difficult to measure, varying the parameter around a reasonable range does not change our key quantitative results. The parameter η is within the range of panel and cross-sectional estimates from Griliches (1990). Since the variety of intermediate goods can be interpreted as the stock of R&D (a directly observable quantity), we can then interpret one

[8] See, for example, Jermann (1998), Croce (2008), Kaltenbrunner and Lochstoer (2008) or Fernandez-Villaverde, Koijen, Rubio-Ramirez and van Binsbergen (2011) for estimation evidence.

[9] To provide further discipline on the calibration of ρ, note that since the ENDO model implies the TFP decomposition, $\Delta z_t = \Delta a_t + \Delta n_t$, we can project log TFP growth on log growth of the R&D stock to back out the residual Δa_t. The autocorrelations of the extracted residual $\Delta \hat{a}_t$ show that we cannot reject that it is white noise. Hence, in levels, it must be the case that a_t is a persistent process to be consistent with this empirical evidence. In our benchmark calibration, the annualized value of ρ is .95.

minus the survival rate ϕ as the depreciation rate of the R&D stock. Hence, we set ϕ to 0.9625 which corresponds to an annualized depreciation rate $1-\phi$ of 15% which is a standard value and assumed by the BLS in the the R&D stock calculations. The scale parameter χ is used to help match balanced growth evidence and set at a value of 0.332.

We calibrate the exogenous growth model (EXO) to facilitate direct comparison with our benchmark model. To do so, we set a trend growth parameter μ equal to 1.90% to match average output growth and adjust the volatility of the forcing process to match the volatility of consumption growth of the benchmark model.

4.1.1 Long-Run Dynamics

Table 2 reports quantitative implications of the model for long-run economic performance. The benchmark model is quantitatively in line with the average growth rate of the economy and the long-run components σ^{LR} of output, consumption and investment volatility, as targeted by our calibration. Two observations are nevertheless noteworthy.

First, the exogenous growth counterpart, while similarly calibrated, generates counterfactually small long-run movements in quantities. This finding reflects the absence of a strong propagation mechanism, which generates endogenous persistence, exhibited by workhorse models in real business cycle tradition. This propagation mechanism will be discussed in section 4.4 below.

Second, while at business cycle frequencies investment growth is much more volatile than both consumption and output growth, in the long run, it is actually smoother. This suggests that movements at higher frequencies are driven by a different set of shocks. Our model of endogenous growth is most readily thought of as theory of long-run movements and therefore we focus on innovations to productivity, to which we turn to now.

4.2 Productivity Dynamics

Many of the key implications of the benchmark model can be understood by looking at the endogenous dynamics of total factor productivity (TFP) growth, ΔZ_t, which we outlined in section

3:

$$E_t[\Delta Z_{t+1}] \approx \vartheta_t \cdot \widehat{S}_t + \phi$$

where $\widehat{S}_t \equiv \frac{S_t}{N_t}$ is the R&D intensity. Therefore, qualitatively, the dynamics of TFP are driven by endogenous movements in R&D.

Quantitatively, the implications of the model will thus depend on the ability of our calibration to match basic stylized facts about R&D activity and innovation. As table 3 documents, the model is broadly consistent with volatilities and autocorrelations of R&D investment, the stock of R&D and R&D intensities in the data. Crucially, as in the data, the R&D intensity is a persistent process and we match its annual autocorrelation of 0.93.

The above decomposition of the expected growth rate of TFP therefore suggests a highly persistent component in TFP growth. Table 4 confirms this prediction, both in the data as well as in the model. While uncovering the expected growth rate of productivity as a latent variable in the data (as in Croce (2008)) suggests an annual persistence coefficient of 0.93, our model closely matches this number with a persistence coefficient of 0.95. Moreover, the volatilities of expected TFP growth rates in the data and in the model roughly match. Note that in contrast to our benchmark model, the EXO specification implies that TFP growth is roughly i.i.d., which is inconsistent with the empirical evidence.

Qualitatively, the above decomposition and the persistence of R&D intensity suggests that R&D intensity should track productivity growth rather well. Figures 1 and 2 visualize these patterns in the model, using a simulated sample path, as well as in the data. The plots visualize the small, but persistent component in TFP growth induced by equilibrium R&D activity.

From an empirical point of view, these results suggest that R&D activity, and especially R&D intensity should forecast productivity growth rather effectively. We confirm this prediction in table 6, which documents results from projecting productivity growth on R&D intensity or R&D growth, respectively, over several horizons. In the data, R&D intensity and growth forecasts productivity growth over several years significantly and the R^2's are increasing with the horizon. Qualitatively, the model replicates this pattern rather well.

The intuition for these results comes from the endogenous R&D dynamics generated by the model. This can be readily gleaned from the impulse responses to an exogenous productivity shock displayed in figure 4, which displays the responses of quantities in the patents sector. Crucially, after a positive shock, profits rise persistently. Intuitively, a positive shock in the final goods sector raises the demand X_t for intermediate goods, and with $\Pi_t = (\nu - 1)X_t$, this translates directly into higher profits. Naturally, given persistently higher profits, the value of a patent goes up, as shown in the third panel. Then, in turn, as the payoff to innovation is the value of patents, this triggers a persistent increase in the R&D intensity. This yields the persistent endogenous component in productivity growth displayed above. Crucially, the exogenous shock has two effects. It immediately raises productivity of the final output firm temporarily (due to the mean-reverting nature of the shock), leading to standard fluctuations at business cycle frequency. In addition, it also induces more R&D which will be reflected in the creation of more patents which has a permanent effect on the level of productivity. Moreover, the increases in R&D are persistent, leading to fluctuations at lower frequencies. Intuitively, in this setting, an exogenous shock to the level of productivity endogenously generates a persistent shock to the growth rate of the economy, or in other words, it generates growth waves.

4.3 Consumption Dynamics and Endogenous Long-Run Risk

In the previous section, we documented that the benchmark model has rich implications for the dynamics of measured TFP, which will naturally be reflected in the quantity dynamics of our production economy. With a view towards asset pricing, we focus on the implications for consumption dynamics in this section. In particular, we examine the dynamics of expected consumption growth that the model generates. While Bansal and Yaron (2004) have shown in an endowment economy that persistent variation in expected consumption growth coupled with recursive preferences can generate substantial risk premia in asset markets, the empirical evidence regarding this channel is still controversial. In this light, providing theoretical evidence in production economies supporting the mechanism would be reassuring [10].

[10]While several papers have considered how such long run risks can arise endogenously in production economies (Croce (2008), Kaltenbrunner and Lochstoer (2008), Campanale, Castro and Clementi (2008)), these studies operate in versions of the real business cycle model (proxied by the EXO specification here)

Table 7 documents basic properties of consumption growth in the model. While the model matches the volatility of consumption growth, it also roughly replicates its annual autocorrelation. This is in sharp contrast to the EXO specification, where consumption growth barely exhibits any significant autocorrelation. More importantly, the table also documents that the benchmark model produces substantial variation in expected consumption growth, and considerably more than the EXO specification. Similarly, this is also reflected in the substantial long-term volatilities that consumption growth exhibits in the model. In line with the asset pricing literature, we will refer to the volatility of consumption growth as business cycle or short-run risk and persistent variation in expected consumption growth as long-run risk. This suggests that the benchmark model generates quantitatively significant long-run risks in consumption growth.

Note that while Bansal and Yaron (2004) (in an endowment economy setting) and Croce (2008) (in a production economy setting) exogenously specify long-run risks by introducing independent, persistent shocks to consumption and productivity growth respectively, in our model fluctuations in realized consumption growth and expected consumption growth are driven by only one source of exogenous uncertainty. Hence, the model translates this disturbance into substantial low-frequency movements in consumption growth, or, in other words, provides a strong endogenous mechanism to propagate this shock. Accordingly, fluctuations in realized consumption growth are closely related to fluctuations in expected consumption growth.

Table 8 reports long-horizon autocorrelations of consumption growth both in the data and in the model. We restrict the empirical sample to 1953 to 2008, to ensure consistency with the availability of R&D data. While our model matches the first autocorrelation of consumption growth almost exactly, the second and third autocorrelation are negative in the data and positive in the model, and more importantly, outside the 95% confidence interval. On the other hand, all longer horizon autocorrelations are within that confidence interval. In short, the consumption dynamics generated by the benchmark model are broadly consistent with the data.

In order to quantify the persistence in consumption growth in the model, we now compute the expected consumption growth process. We do so in two ways. In the first method, we take the consumption growth policy function from the numerical solution and directly take conditional ex-

and typically do not generate sufficient endogenous risks to match asset market statistics

pectations to obtain the expected consumption growth policy. Then we can directly simulate the process using this function. In the second method, we first simulate log consumption growth Δc_t as well as the state variables log capital-to-R&D-capital ratio $\widehat{k}_t$, and log productivity shock a_t from the model and proceed by running the following regression $\Delta c_{t+1} = \beta_0 + \beta_1 \widehat{k}_t + \beta_2 a_t + \epsilon_{c,t+1}$ so that the fitted values from this regression give the expected consumption growth process. Table 9 reports the results. The two methods yield practically the same process. This is not surprising as consumption growth in the model is approximately log-linear in the state variables. Also the endogenous expected consumption growth dynamics generated from our model are roughly similar to the exogenous specification (x_t) from Bansal and Yaron (2004). In particular, our process is slightly more persistent but slightly less volatile than the one in Bansal and Yaron.

Naturally, persistence in expected consumption growth is just a reflection of persistent dynamics in productivity growth. Empirically, this suggests that measures related to innovation, and the R&D intensity and R&D growth in particular, should have forecasting for consumption growth. We verify this in table 10, which reports results from projecting future consumption growth over various horizons on the R&D intensity and growth. Empirically, these innovation measures predict future consumption growth over horizons up to 5 years with significant point estimates and R^2's are increasing with the horizon. Qualitatively, the model reflects this pattern reasonably well. This gives empirical support to the notion of innovation-driven low-frequency variation in consumption growth.

4.4 Fluctuations and Propagation

While consumption dynamics are important for asset pricing, endogenous persistent variation in expected productivity growth suggests a propagation mechanism for quantities that standard macro models typically lack. We therefeore turn to a more systematic discussion of the macroeconomic implications of the model.

Table 11 reports standard business cycle statistics implied by the model. While the model is calibrated to replicate long-run dynamics of the aggregate economy, the table shows that it is also reasonably consistent with basic business cycle statistics. In particular, our benchmark model does

just as well as the EXO model, which is essentially a version of a standard real business cycle model, meaning that the ENDO model generates high-frequency dynamics in line with the canonical real business cycle model. On the other hand, all specifications predict investment to be too smooth. This is because the model is calibrated to generate realistically smooth long-run investment dynamics, suggesting that different shocks drive investment volatility at business cycle frequencies [11]. Looking beyond the standard business cycle statistics, the macroeconomic implications of our benchmark model and the exogenous growth counterpart are quite different, as we now explore.

Table 12 reports autocorrelations of basic growth rates in the data, the ENDO, and the EXO model. Note first that while all growth rates exhibit considerable positive autocorrelation at annual frequencies, the corresponding persistence implied by the EXO models is virtually zero, and sometimes even negative. This is one of the main weaknesses of the real business cycle model (as pointed out e.g. in Cogley and Nason (1995)). In stark contrast, our ENDO model generates substantial positive autocorrelation in all quantities, and in general are quantitatively close to their data counterparts. Note that the exogenous component of productivity is the same in both model. Accordingly, the ENDO model possesses a strong propagation mechanism induced by the endogenous component of productivity, e.g. by R&D.

The intuition for this endogenous propagation is of course simple and tightly linked to the dynamics of TFP documented in the previous section. To the extent that innovation induces a persistent component in productivity, this will be reflected in quantity dynamics. Recall however, that the TFP dynamics implied by the model are consistent with the empirical evidence. As for consumption growth, this suggests that the drivers of expected productivity growth, namely R&D intensity and R&D growth, should forecast aggregate growth rates. This is verified in table 13 for output growth.

The propagation mechanism implies that macroeconomic quantities display markedly different behavior at different frequencies. In other words, it implies a rich intertemporal distribution of growth rates. The results in table 12 also suggest that the implied volatilities of growth rates of the EXO and ENDO models are basically undistinguishable at short horizons, however in the ENDO model

[11] Similarly, we abstract from endogenous movements in labor supply, as those mostly drive fluctuations at business cycle frequencies.

the volatilities increase fairly quickly over longer horizons. Essentially, the ENDO model generates significant quantity fluctuations at lower frequencies, while the EXO model does not.

Another implication of the model is that it generates cash flow dynamics that are in line with the empirical evidence. First of all, it generates strongly procyclical profits. This can be seen from figure 4. This is in line with recent work on expanding variety models in Bilbiie, Ghironi, Melitz (2007), but typically presents a challenge for macro models. In our setting, procyclical profits driven by the procyclical demand for intermediate goods. Second, the model generates a persistent component in dividend growth. This can be seen in table 15, which documents considerable volatility in conditional expected dividend growth, which implies substantial variation in the conditional mean of cash flow growth. This is visualized in figure 6. Again, this is in stark contrast to the exogenous growth specification. This will be important from an asset pricing perspective, as only the benchmark model generates sufficient long-term uncertainty about dividend growth.

4.5 Asset Pricing Implications

The productivity dynamics in the model and the resulting endogenous persistence in consumption and cash flows generate sizeable risk premia in asset markets, as we now document. Endogenous persistence in growth rates affects asset prices in our model, because when agents have Epstein-Zin utility with a preference for an early resolution of uncertainty, not only are innovations to realized consumption and dividend growth priced, but also innovations to expected consumption and dividend growth.

Consistent with the multi-sector structure of our model, the stock market is a claim to the net payout from production; equation (18) provides a decomposition of the value of this claim into the value of physical capital and patents, hence intangible capital. Accordingly, we can separately define the returns on physical capital, the return on intangible capital, and the spread between the two. We will suggestively relate that spread to the value premium, the return spread between high book-to-market stocks (value stocks) and low book-to-market stocks (growth stocks). The link is more suggestive as growth firms in the data likely are intangibles intensive but also hold physical capital, while in our model they do not, and likewise for value firms.

Table 14 reports asset market statistics, for the benchmark model and alternative specifications.

Quantitatively, the benchmark model generates a sizeable excess return on stocks of close to 3%, a premium on physical capital in excess of 4%, a value spread close to the excess return on the aggregate stock market, plus a low and smooth risk free rate. The volatility of the aggregate stock market returns is close to 5%. The volatilities of the return on physical capital and the value spread are of considerable magnitude as well.

While sizeable, the premia and volatilities of returns in the model do not rationalize their empirical counterparts entirely. In line with our interpretation of the benchmark endogenous growth model as a model of long-run dynamics, we view the model implied premia and volatilities as those components reflecting uncertainty about long-term growth prospects and productivity. As documented earlier, the benchmark model is calibrated to match such long-run risks in the language of Bansal and Yaron, while it does not generate realistic business cycle or short-run risks, such as investment volatility. Indeed, Ai, Croce and Li (2010) report that empirically the productivity-driven fraction of return volatility is just around 6%, which is roughly consistent with our quantitative finding. On the other hand, table 14 also reports the asset pricing implications of a version of the endogenous growth model which is calibrated to match short-run consumption risks in a long-sample starting from the great depression. This calibration produces an overall equity premium of close to 6%, and a value premium of a similar magnitude.

In order to understand these results, it is instructive to compare the asset pricing implications of the benchmark model with those of the exogenous growth specification. To facilitate comparison, we focus on the returns on physical capital in the following discussion. While as discussed previously, the quantity implications of the models are similar at high-frequencies, the pricing implications are radically different. As can be seen from the table, the risk free rate is counterfactually high in the exogenous growth specifications, and the equity premium is close to zero and only a tiny fraction of what obtains in the benchmark model. These differences are intimately connected to differences in low-frequency dynamics that the two models generate. Intuitively, in the settings with exogenous growth, expected growth rates are roughly constant (as in the real business cycle model), therefore diminishing households' precautionary savings motive. In such a setting, households want to borrow against their future income, which in equilibrium can only be prevented by a prohibitively high interest rate. In the endogenous growth setting, however, taking advantage of profit opportunities

in the intermediate goods sector leads to long and persistent swings in aggregate growth rates, and higher volatility over longer horizons. In this context, households optimally save for low growth episodes, leading to a lower interest rate in equilibrium.

Moreover, in contrast to intangible capital, the claim to physical capital is very risky in the model. This suggests that physical capital is endogenously more exposed to long-run uncertainty. The reason is twofold. First, as discussed above, the model generates endogenous long-run risks in consumption growth reflected in the stochastic discount factor. Second, the level of the risk premium also implies that in equilibrium, dividends on physical capital are risky. The reason is that these dividends naturally inherit a persistent component from the endogenous component of productivity. These cash flow dynamics not only affect risk premia, but naturally, also asset market valuations, as documented in figure 6. Specifically, the figure documents that following a productivity shock expected growth rates respond strongly in a persistent fashion in the ENDO model whereas in the EXO model expected growth rates are virtually unresponsive to the shock. In particular, expected dividend growth rates endogenously exhibit substantial persistent variation consistent with the setup in Barsky and DeLong (1993), who show that such a process can explain long swings in stock markets, and in Bansal and Yaron (2004).

The impulse responses also show that innovations to realized consumption and dividend growth are tightly linked to innovations to expected growth. Both of these innovations are priced when agents have Epstein-Zin utility with a preference for early resolution of uncertainty. In this case, agents fear that persistent slowdowns in growth coincide with a fall in asset prices. Therefore bad shocks are simultaneously bad shocks for the long run, which renders equity claims very risky.

Figure 7 illustrates this. In the benchmark model the response of the stochastic discount factor is substantially larger on impact than in the exogenous growth counterpart as a shock to realized consumption growth leads to a revision in growth expectations which is picked up in the stochastic discount factor as a revision to expected continuation utility. This is in contrast to the exogenous growth specification, where consumption growth is essentially iid. Moreover, the benchmark model displays stronger co-movements between returns and the discount factor, which leads to a higher risk premium since $E[r_d - r_f] \approx -cov(m, r_d)$.

4.6 Asset Prices and Growth

While the endogenous growth rate dynamics in the model in conjunction with recursive preferences help explain large risk premia in the data, asset prices also have important feedback effects on the macroeconomy. In particular, realistic risk premia in the model foster growth and amplify long-run movements in growth rates, a phenomenon we label as long-run amplification. This is in contrast to real business cycle models, in which risk and risk premia do not affect quantity dynamics, a point which was forcefully made by Tallarini (2000). Formally, these feedback effects can be traced to equation (26) which relates the growth rate of the economy to discount rates and profit opportunities in the intermediate goods sector. Our model suggests that such a feedback channel can be quantitatively significant.

Table 15 provides quantitative evidence on long-run amplification. It reports the volatilities of conditional means, long-run risks in other words, of various quantities. It does so for the benchmark model, the exogenous growth model, and a version of the endogenous growth model solved with CRRA preferences by setting the IES to the inverse of risk aversion. Not surprisingly, movements in conditional means are much more pronounced in the benchmark model relative to the exogenous growth model. Notably, however, the CRRA case of the endogenous growth model barely generates movements in conditional means. Thus, in our benchmark model, realistic asset price implications provide long-run amplification. Recursive preferences in conjunction with endogenous persistent fluctuations in growth rates increase the volatility of asset prices. Incentives to innovate reflect prices however, which renders innovation more volatile and amplifies long-run movements in growth.

We provide quantitative evidence on average growth rate effects in table 16, where we report sensitivity of model implications with respect to the key preference parameters, risk aversion and intertemporal elasticity of substitution. Consider first varying risk aversion, in the first 2 columns. Consistent with the results in Tallarini (2000), varying risk aversion barely affects standard business cycle statistics, that is, second moments. In other words, while varying risk aversion does not affect the amount of risk in the economy, it affects the price of risk and risk compensation, reflected in substantial differences in risk premia. Therefore, relative to Tallarini, the benchmark endogenous growth model exhibits a new effect, namely sensitivity of the average growth rate relative to the

risk aversion. Specifically, raising risk aversion fosters growth. This has a simple intuition: Higher compensation for the same risks, or similarly, higher price for the same magnitude of risks reflected by in a higher Sharpe ratio, makes investment in risky assets more attractive and therefore channels resources towards innovation and R&D. This is reflected in higher R&D investment, as measured by the R&D intensity, and hence higher growth.

In the last two columns, we keep risk aversion fixed at the benchmark level, but vary the intertemporal elasticity of substitution. Note that for all specifications we have $\psi > \frac{1}{\gamma}$, so that irrespective of the specification, agents have a preference for early resolution of uncertainty. Varying the IES changes the amount of risk in the economy, and its intertemporal distribution. Raising the IES is akin to increasing the propensity to substitute over time, which increases the response of investment to productivity and expected productivity growth and accordingly the respective volatilities. In turn this smoothes consumption growth and increases its persistence. This raises the volatility of the conditional mean of consumption growth. Raising the IES therefore reduces short-run risk and increases long-run risk, while lowering the IES increases short-run risk and reduces long-run risk. With a high price of long-run risk, the net effect is an increase of the risk premium in the first case, and a fall in the latter case. As above, the average growth rate of the economy is increasing in the Sharpe ratio.

4.7 Long-Term Comovement

Our model also has realistic implications for comovement between prices and quantities at lower frequencies. In the following we identify low frequency movements in growth rates using a bandpass filter which isolates movements at frequencies between 32 and 100 quarters.

Figure 8 reveals that the model replicates the low-frequency comovements between productivity and quantities in the data. This is noteworthy because it reveals the significant variation macro data exhibit at lower frequencies and the significant comovement between productivity and quantities, which is mirrored by the ENDO model.

Figure 9 shows the close match between the price-dividend ratio and productivity growth in the data and the benchmark model at low-frequencies. This strongly suggests productivity-driven slow movements in asset market valuations in the data. In the model, these movements are driven by

variation in expected cash flows, induced by time variation in R&D intensity. The long swings in price-dividend ratios are consistent with the evidence in Barsky and Delong (1993).

At lower frequencies we also find strong cross-correlations between stock returns and consumption growth. This is displayed in figure 10, indicating the lag-lead structure between returns and consumption growth. In the data and at low frequencies, returns lead consumption growth by several quarters and the lead correlations die away more slowly (relative to the lag correlations). In other words, lower-frequency movements in returns contain important information regarding long-run movements in future growth. The ENDO model replicates this feature whereas the EXO model does not. This important divergence between the two models is due to the fact that in the ENDO model, growth rates contain a predictable component, which is absent in the EXO models, that is a key determinant of asset prices. In sum, the benchmark model is able reconcile the long-term relationship between returns and growth that the neoclassical growth model fails to produce.

5 Conclusion

Starting from the notion that asset prices reflect expectations about future growth, we provide a quantitative analysis of a production economy whose long-term growth prospects are endogenously determined by innovation and R&D. By integrating innovation and R&D into a real business cycle model with recursive preferences, our model constitutes a straightforward and highly tractable extension of the workhorse model of modern macroeconomics. In sharp contrast to the latter, however, our baseline model jointly rationalizes key features of asset returns and long-run macroeconomic performance in the data.

In the model, favorable economic conditions boost innovation and the development of new technologies. Since technological progress fosters long-run economic growth, endogenous innovation generates a powerful propagation mechanism for shocks reflected in persistent variation in long-term growth prospects. With recursive preferences, innovations to expected growth are priced and lead to high risk premia in asset markets, as agents fear that persistent slowdowns in growth coincide with low asset valuations. Formally, we show that R&D drives an endogenous predictable component in measured productivity, which gives an innovation-based explanation of long-run pro-

ductivity risk in the data.

Our model thus allows to empirically identify economic sources of long-run risks. Indeed, we document novel empirical evidence that measures of innovation have significant predictive power for aggregate growth rates at longer horizons.

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Appendix A. Data

Annual and quarterly data for consumption, capital investment, and GDP are from the Bureau of Economic Analysis (BEA). Annual data on private business R&D investment is from the survey conducted by National Science Foundation (NSF). Annual data on the stock of private business R&D is from the Bureau of Labor Statistics (BLS). Annual productivity data is obtained from the BLS and is measured as multifactor productivity in the private nonfarm business sector. The sample period is for 1953-2008, since R&D data is only available during that time period. Consumption is measured as expenditures on nondurable goods and services. Capital investment is measured as private fixed investment. Output is measured as GDP. The variables are converted to real using the Consumer Price Index (CPI), which is obtained from the Center for Research in Security Prices (CRSP). Monthly nominal return and yield data are from CRSP. The real market return is constructed by taking the nominal value-weighted return on the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) and deflating it using the CPI. The real risk-free rate is constructed by using the nominal average one-month yields on treasury bills and taking out expected inflation.[12] Aggregate market and book values of assets are from the Flow of Funds account.

[12]We model the monthly time series process for inflation using an AR(4).

Table 1: Calibration

Parameter	Description	ENDO	EXO
β^4	Subjective Discount Factor	0.984	0.984
ψ	Elasticity of Intertemporal Substitution	1.85	1.85
γ	Risk Aversion	10	10
ξ	Intermediate Goods Share	0.5	-
ν	Elasticity of Substitution Between Intermediate Goods	1.65	-
α	Capital Share	0.35	0.35
ρ^4	Autocorrelation of Ω	0.95	0.95
χ	Scale Parameter	0.332	-
ϕ	Survival Rate of Intermediate Good	0.9625	-
η	Elasticity of New Intermediate Goods wrt R&D	0.83	-
δ	Depreciation Rate of Capital Stock	0.02	0.02
σ	Volatility of Productivity Shock ϵ	1.75%	0.97%
ζ	Elasticity of Capital Investment Rate	0.70	0.70
$\mu * 4$	Trend Growth Rate	-	1.90%

This table reports the benchmark quarterly calibration used for the endogenous growth (ENDO) and exogenous growth (EXO) models.

Table 2: Long-Run Dynamics

	Data	ENDO	EXO
$E[\Delta y]$	1.90%	1.90%	1.90%
$\sigma^{LR}_{\Delta y}$	0.24%	0.22%	0.13%
$\sigma^{LR}_{\Delta c}$	0.28%	0.24%	0.15%
$\sigma^{LR}_{\Delta i}$	0.18%	0.17%	0.09%

This table reports the average growth rate as well as annualized volatilities of long-run components of output, consumption and investment growth from the data and from the ENDO and EXO models. The bandpass filter from Christiano and Fitzgerald (2003) is used to isolate the components of the various frequencies. We identify long-run components with frequencies of 100 to 200 quarters. Output, consumption and investment data are from the BEA.

Table 3: Innovation Dynamics

	Data	ENDO
$\sigma_{\Delta s}$	4.89%	3.82%
$AC1(\Delta s)$	0.21	0.06
$AC1(\Delta n)$	0.90	0.94
$AC1(S/N)$	0.93	0.93

This table reports summary statistics for innovation-related variables: log R&D growth, log stock of R&D growth and R&D intensity. The first column presents the statistics from the data and the second column is from the endogenous growth model (ENDO). The models are calibrated at a quarterly frequency and then growth rates are time-aggregated to an annual frequency to compute the autocorrelations. R&D stock data are from the BLS. R&D flow data are from the NSF.

Table 4: Expected Productivity Growth Dynamics

	Estimate	ENDO
$\rho_{\tilde{x}}$	0.93	0.95
$\sigma(\tilde{x})$	1.10%	1.20%

This table reports the annual persistence and standard deviation of the expected growth rate component of productivity growth from the data and from the endogenous growth (ENDO) model. The estimates are taken from Croce (2010), where the expected growth rate component of productivity $\tilde{x}_{t-1}$ is a latent variable that is assumed to follow an AR(1). In contrast, in the ENDO model the expected growth rate component is the growth rate of the variety of intermediate goods Δn_t, a endogenous structural variable of the model. In particular, since the shock Ω_t is persistent, log productivity growth can be written approximately as $\Delta z_t = \tilde{x}_{t-1} + \epsilon_t$, where $\tilde{x}_{t-1} \equiv \Delta n_t$ and ϵ_t is an iid disturbance. The ENDO model endogenously generates a productivity process that is the same as the exogenous specification of Croce (2010), which is supported empirically.

Table 5: Productivity Growth Dynamics

	Data	ENDO	EXO
$AC1(\Delta z)$	0.09	0.11	-0.02
$\sigma(E_t[\Delta z_{t+1}])$		0.38%	0.15%
$\sigma_{\Delta z(5)}$		9.29%	4.15%
$\sigma_{\Delta z(10)}$		15.79%	5.55%
$\sigma_{\Delta z(20)}$		25.24%	6.86%

This table reports summary statistics for productivity growth: Annual autocorrelation, volatility of the conditional mean, and 5, 10 and 20 year volatilities. The first column presents the statistics from the data, the second column is from the endogenous growth model (ENDO), and the last column from the exogenous growth model (EXO). The models are calibrated at a quarterly frequency and then growth rates are time-aggregated to an annual frequency to compute the autocorrelations. Annual multifactor productivity data are from the BLS.

Table 6: Productivity Growth Forecasts

Forecasts with R&D Intensity					
Horizon (k)	Data			ENDO	
	β	S.E.	R^2	β	R^2
1	0.014	0.009	0.031	0.075	0.039
2	0.031	0.015	0.080	0.142	0.062
3	0.049	0.024	0.120	0.204	0.077
4	0.069	0.032	0.174	0.261	0.088
5	0.091	0.041	0.232	0.314	0.095
Forecasts with R&D Growth					
Horizon (k)	Data			ENDO	
	β	S.E.	R^2	β	R^2
1	0.431	0.190	0.113	0.560	0.037
2	0.820	0.315	0.192	1.070	0.060
3	1.230	0.452	0.262	1.533	0.076
4	1.707	0.522	0.376	1.948	0.084
5	2.092	0.599	0.444	2.322	0.090

This table presents annual productivity growth forecasting regressions from the data and from the benchmark endogenous growth model (ENDO) for horizons (k) of one year to five years. Specifically, log productivity growth is projected on log R&D intensity, $\Delta z_{t,t+1} + \cdots + \Delta z_{t+k-1,t+k} = \alpha + \beta \hat{s}_t + \nu_{t,t+k}$ (first panel) and on log R&D stock growth, $\Delta z_{t,t+1} + \cdots + \Delta z_{t+k-1,t+k} = \alpha + \beta \Delta n_t + \nu_{t,t+k}$ (second panel). In the data the regression is estimated via OLS with Newey-West standard errors with $k-1$ lags. The model regression results correspond to the population values. Overlapping annual observations are used. Multifactor productivity and R&D stock data is from the BLS, and R&D flow data is from the NSF.

Table 7: Consumption Dynamics

	Data	ENDO	EXO
$\sigma_{\Delta c}$	1.42%	1.42%	1.42%
$AC1(\Delta c)$	0.40	0.39	-0.002
$\sigma(E_t[\Delta c_{t+1}])$		0.51%	0.09%
$\sigma_{\Delta c(5)}$		6.63%	3.14%
$\sigma_{\Delta c(10)}$		11.97%	4.30%
$\sigma_{\Delta c(20)}$		21.18%	5.58%

This table reports summary statistics for consumption growth: Annual volatility, annual autocorrelation, volatility of the conditional mean, and 5, 10 and 20 year volatilities. The first column presents the statistics from the data, the second column is from the endogenous growth model (ENDO), and the last column from the exogenous growth model (EXO). The models are calibrated at a quarterly frequency and then growth rates are time-aggregated to an annual frequency to compute the autocorrelations. Annual consumption data are from the BEA.

Table 8: Consumption Autocorrelations

	Data	ENDO	95% confidence interval	
			lower	upper
$AC1(\Delta c)$	0.40	0.39	0.05	0.53
$AC2(\Delta c)$	-0.09	0.26	-0.01	0.49
$AC3(\Delta c)$	-0.17	0.21	-0.06	0.46
$AC4(\Delta c)$	-0.11	0.17	-0.11	0.42
$AC5(\Delta c)$	0.06	0.13	-0.15	0.39
$AC6(\Delta c)$	0.10	0.11	-0.17	0.38
$AC7(\Delta c)$	-0.02	0.09	-0.20	0.35
$AC8(\Delta c)$	-0.16	0.05	-0.24	0.33
$AC9(\Delta c)$	-0.17	0.03	-0.25	0.32
$AC10(\Delta c)$	-0.01	0.02	-0.27	0.31

This table reports long-horizon autocorrelations of consumption growth. The first column presents the statistics from the data for the sample 1953-2008 , the second column is from the endogenous growth model (ENDO), with lower and upper boundaries of the 95% confidence interval. Model estimates are obtained from 200 simulations of 56 years of data at quarterly frequency, time-aggregated to annual frequency. Annual consumption data are from the BEA.

Table 9: Expected Consumption Growth Dynamics

	BY	Method 1	Method 2
ρ	0.979	0.981	0.981
$\tilde{\sigma_x}$	0.12%	0.10%	0.10%

We fit the the expected consumption growth process from our model to an AR(1) process $x_t = \rho_x x_{t-1} + \sigma_x \epsilon_{x,t}$, where $\epsilon_{x,t} \sim N(0,1)$. This table reports the persistence parameter ρ_x and annualized volatility parameter for the benchmark ENDO model using the two methods and compares them to values from Bansal and Yaron (2004). For monthly data, $\tilde{\sigma_x} \equiv \sigma_x \times \sqrt{12}$. For quarterly data, $\tilde{\sigma_x} \equiv \sigma_x \times \sqrt{4}$. The simulation length is for 1,000,000 quarters.

Table 10: Consumption Growth Forecasts

Forecasts with R&D Intensity						
Horizon (k)	Data			ENDO		
	β	S.E.	R^2	β	S.E.	R^2
1	0.017	0.006	0.070	0.068	0.028	0.141
2	0.034	0.012	0.105	0.118	0.052	0.161
3	0.048	0.017	0.131	0.168	0.073	0.179
4	0.062	0.023	0.163	0.200	0.095	0.175
5	0.077	0.030	0.202	0.224	0.116	0.168
Forecasts with R&D Growth						
Horizon (k)	Data			ENDO		
	$\hat{\beta}$	S.E.	R^2	β	S.E.	R^2
1	0.217	0.084	0.094	0.573	0.189	0.158
2	0.395	0.178	0.115	1.012	0.356	0.189
3	0.540	0.276	0.132	1.437	0.526	0.207
4	0.703	0.347	0.168	1.750	0.704	0.203
5	0.842	0.401	0.198	1.993	0.878	0.193

This table presents annual consumption growth forecasting regressions from the data and from the benchmark endogenous growth model (ENDO) for horizons (k) of one year to five years. Specifically, real consumption growth is projected on log R&D intensity, $\Delta c_{t,t+1} + \cdots + \Delta c_{t+k-1,t+k} = \alpha + \beta \hat{s}_t + \nu_{t,t+k}$ (first panel) and log R&D stock growth, $\Delta c_{t,t+1} + \cdots + \Delta c_{t+k-1,t+k} = \alpha + \beta \Delta n_t + \nu_{t,t+k}$ (second panel). In the data the regression is estimated via OLS with Newey-West standard errors with $k-1$ lags. The model regression results come from 200 simulated data-equivalent samples. Overlapping annual observations are used. Consumption data is from the BEA, R&D flow data is from the NSF, and R&D stock data is from the BLS.

Table 11: Business Cycle Statistics

	Data	ENDO	EXO
$\sigma_{\Delta c}/\sigma_{\Delta y}$	0.61	0.61	1.13
$\sigma_{\Delta i}/\sigma_{\Delta c}$	4.38	2.23	0.79
$\sigma_{\Delta s}/\sigma_{\Delta y}$	2.10	1.64	-
$\sigma_{\Delta z}/\sigma_{\Delta y}$	1.22	1.52	1.54

This table presents annual second moments from the endogenous growth (ENDO) model, the exogenous growth (EXO) model, and the data. The models are calibrated at a quarterly frequency and the moments are annualized. Annual macro data are obtained from the BEA, BLS, and NSF. The data sample is 1953-2008.

Table 12: First Autocorrelations

	Data	ENDO	EXO
$AC1(\Delta z)$	0.09	0.11	-0.020
$AC1(\Delta c)$	0.40	0.46	-0.002
$AC1(\Delta y)$	0.37	0.21	0.001
$AC1(\Delta i)$	0.25	0.14	0.012
$AC1(Q)$	0.95	0.96	0.89

This table reports first autocorrelations of annual variables. The first column presents the statistics from the data, the second column is from the endogenous growth model (ENDO), and the last column from the exogenous growth model (EXO). The models are calibrated at a quarterly frequency and then growth rates are time-aggregated to an annual frequency to compute the autocorrelations. Annual macro data are from the BEA, BLS, and NSF. Annual market and book values of assets are from the Flow of Funds account.

Table 13: Output Growth Forecasts

Forecasts with R&D Intensity					
Horizon (k)	Data			ENDO	
	$\hat{\beta}$	S.E.	$\hat{R}^2$	β	R^2
1	0.020	0.013	0.040	0.085	0.105
2	0.046	0.022	0.084	0.163	0.161
3	0.068	0.029	0.119	0.236	0.195
4	0.089	0.041	0.158	0.306	0.217
5	0.114	0.051	0.210	0.372	0.231
Forecasts with R&D Growth					
Horizon (k)	Data			ENDO	
	β	S.E.	R^2	β	R^2
1	0.267	0.130	0.061	0.635	0.120
2	0.453	0.261	0.067	1.230	0.159
3	0.572	0.387	0.073	1.780	0.193
4	0.763	0.457	0.113	2.307	0.212
5	0.940	0.499	0.159	2.792	0.222

This table presents annual output growth forecasting regressions from the data and from the benchmark endogenous growth model (ENDO) for horizons (k) of one year to four years. Specifically, real output growth is projected on on log R&D intensity, $\Delta c_{t,t+1} + \cdots + \Delta c_{t+k-1,t+k} = \alpha + \beta \hat{s}_t + \nu_{t,t+k}$ (first panel) and on log R&D stock growth, $\Delta c_{t,t+1} + \cdots + \Delta c_{t+k-1,t+k} = \alpha + \beta \Delta n_t + \nu_{t,t+k}$ (second panel). In the data the regression is estimated via OLS with Newey-West standard errors with $k-1$ lags. The model regression results correspond to the population values. Overlapping annual observations are used. Output data is from the BEA, R&D flow data is from the NSF, and R&D stock data is from the BLS.

Table 14: Asset Pricing Implications

	ENDO	ENDO-HV	EXO
First Moments			
$E[r_f]$	1.21%	1.21%	2.61%
$E[r_m^* - r_f]$	2.92%	5.76%	0.12%
$E[r_d^* - r_f]$	4.10%	8.33%	0.12%
$E[r_d^* - r_{ic}^*]$	3.27%	6.89%	-
Second Moments			
$\sigma_{\Delta c}$	1.42%	2.72%	1.42%
σ_{r_f}	0.30%	0.38%	0.05%
$\sigma_{r_m^* - r_f}$	4.86%	6.73%	2.27%
$\sigma_{r_d^* - r_f}$	7.08%	9.49%	2.27%
$\sigma_{r_d^* - r_{ic}^*}$	5.13%	7.81%	-

This table compares asset pricing implications from alternate calibrations of the endogenous growth model (ENDO), as well as the exogenous growth counterpart. ENDO-HV corresponds to a "high volatility" calibration of the volatility parameter σ to match consumption volatility of the post-great depression sample (1930-2008). The scale parameter χ and the subjective discount factor β are adjusted to match the average output growth rate and risk-free rate from the benchmark model. All other parameters remain the same as in the benchmark calibration. $E[r_m^* - r_f]$ refers to the risk premium on the aggregate stock market, $E[r_d^* - r_f]$ to the risk premium on physical capital (the claim to final good dividends) and $E[r_d^* - r_{ic}^*]$ to the spread between expected returns on physical and intangible capital. The summary statistics are annualized. The risk premiums are levered following Boldrin, Christiano, and Fisher (2001).

Table 15: Volatility of Expected Growth Rates

	ENDO	EXO	ENDO-CRRA
$\sigma(E_t[\Delta z_{t+1}])$	0.38%	0.15%	0.06%
$\sigma(E_t[\Delta y_{t+1}])$	0.42%	0.08%	0.09%
$\sigma(E_t[\Delta i_{t+1}])$	0.37%	0.05%	0.21%
$\sigma(E_t[\Delta d_{t+1}])$	0.92%	0.18%	0.10%

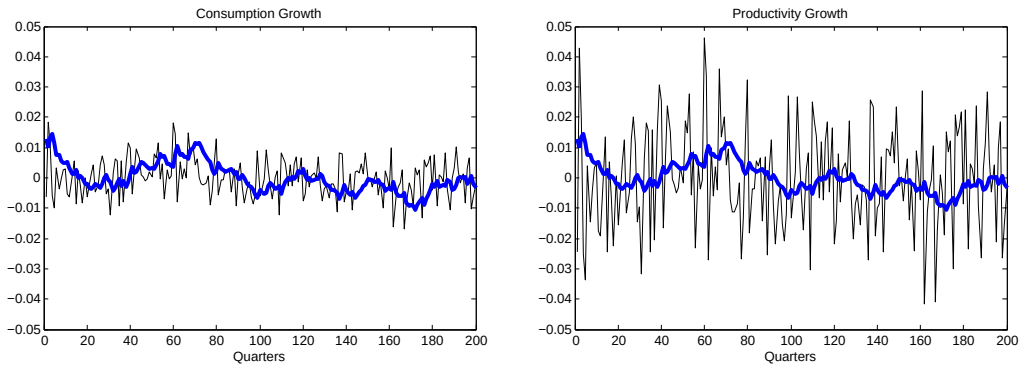
This table reports annualized volatilities of expected growth rates from alternate calibrations of the endogenous growth (ENDO) and exogenous growth (EXO) models. In the ENDO-CRRA specification, the IES is set $\frac{1}{\gamma}$.

Table 16: Sensitivity Analysis: Preference Parameters

	$\gamma = 2$	$\gamma = 15$	$\psi = 0.5$	$\psi = 2.2$
First Moments				
$E[\Delta y]$	1.87%	2.02%	0.86%	2.38%
$E[r_f]$	2.40%	0.50%	2.23%	0.87%
$E[r_m^* - r_f]$	0.68%	6.27%	1.28%	5.06%
$E[S/N]$	0.081	0.084	0.077	0.086
Other Moments				
$\sigma_{\Delta c}/\sigma_{\Delta y}$	0.61	0.61	1.09	0.52
$\sigma_{\Delta i}/\sigma_{\Delta c}$	2.23	2.23	0.57	2.37
$\sigma_{\Delta s}/\sigma_{\Delta y}$	1.64	1.64	1.11	1.73
$\sigma_{\Delta c}$	1.42%	1.42%	2.61%	1.21%
σ_{r_f}	0.30%	0.30%	0.38%	0.27%
$\sigma_{r_m - r_f}$	4.86%	4.86%	3.41%	5.59%
$AC1(\Delta c)$	0.46	0.46	0.07	0.62
$\sigma(E_t[\Delta c_{t+1}])$	0.51%	0.51%	0.19%	0.59%
Sharpe Ratio	0.10	0.88	0.38	0.69

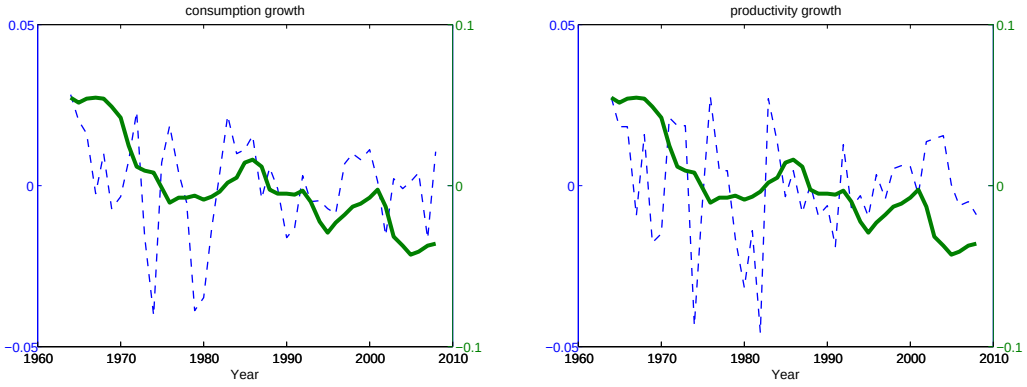
This table compares key summary statistics from alternate calibration of the endogenous growth model (ENDO) that vary the preference parameters, risk aversion γ and the elasticity of intertemporal substitution ψ, one at a time while holding all other parameters fixed at the benchmark calibration. Note that at the benchmark calibration, $\gamma = 10$ and $\psi = 1.85$. The models are calibrated at a quarterly frequency and the summary statistics are annualized. The risk premium is levered following Boldrin, Christiano, and Fisher (2001).

Figure 1: Growth Rates and R&D Intensity



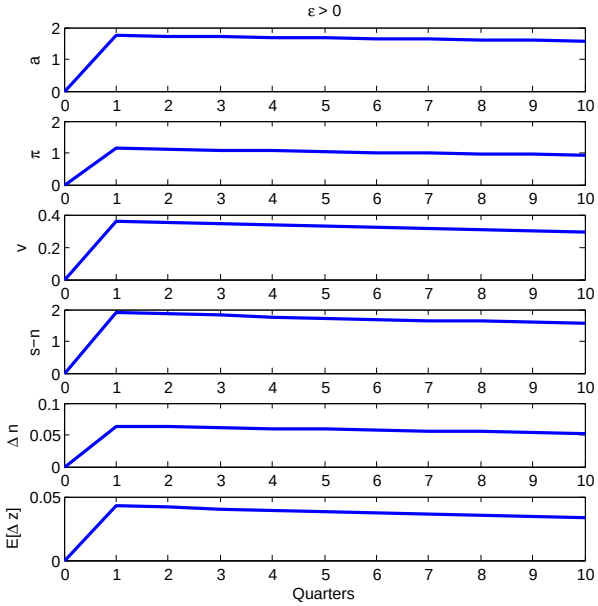
The left panel plots demeaned log consumption growth Δc_t (thin line) with R&D intensity $\frac{S_{t-1}}{N_{t-1}}$ (thick bold line) from the ENDO model for a sample simulation of 200 quarters. The right panel plots demeaned log output growth Δy_t (thin line) with R&D intensity $\frac{S_{t-1}}{N_{t-1}}$ (thick bold line) from the ENDO model for a sample simulation of 200 quarters.

Figure 2: Growth Rates and R&D Intensity from Data



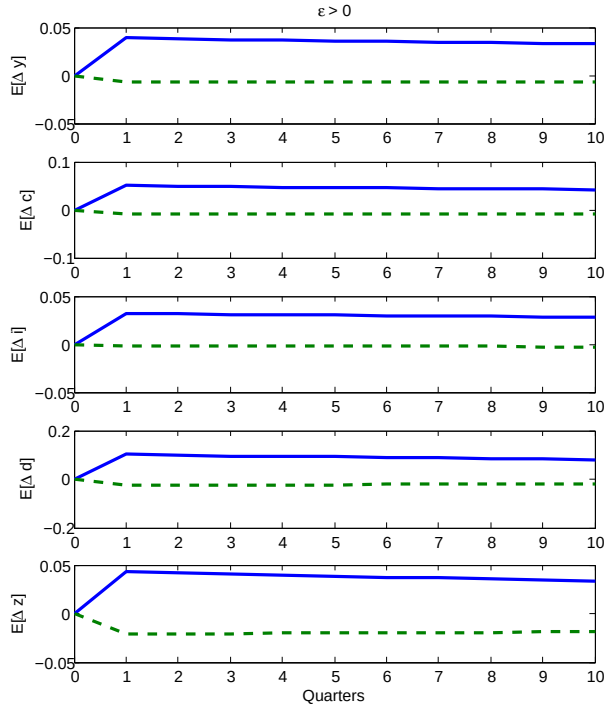
The left panel plots demeaned log consumption growth Δc_t (dashed line) with R&D intensity $\frac{S_{t-1}}{N_{t-1}}$ (bold line) from the data. The right panel plots demeaned log output growth Δy_t (dashed line) with R&D intensity $\frac{S_{t-1}}{N_{t-1}}$ (bold line) from the data. Annual data on aggregate output and consumption is from the BEA. Annual data on R&D expenditures are from the NSF and data on R&D stocks are from the BLS. In the model, R&D intensity is the key determinant of expected growth rates.

Figure 3: Endogenous Growth Mechanism



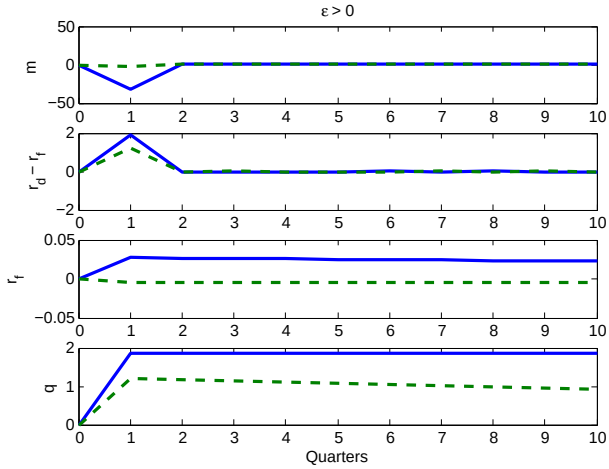
This figure shows quarterly log-deviations from the steady state for the ENDO model. All deviations are multiplied by 100.

Figure 4: Expected Growth Rates



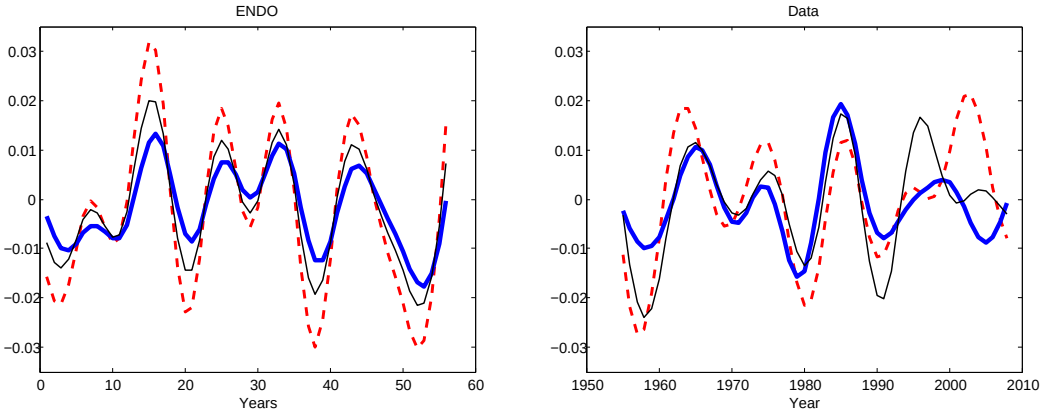
This figure shows quarterly log-deviations from the steady state for the ENDO (solid line) and EXO (dashed line) models. All deviations are multiplied by 100.

Figure 5: Asset Prices



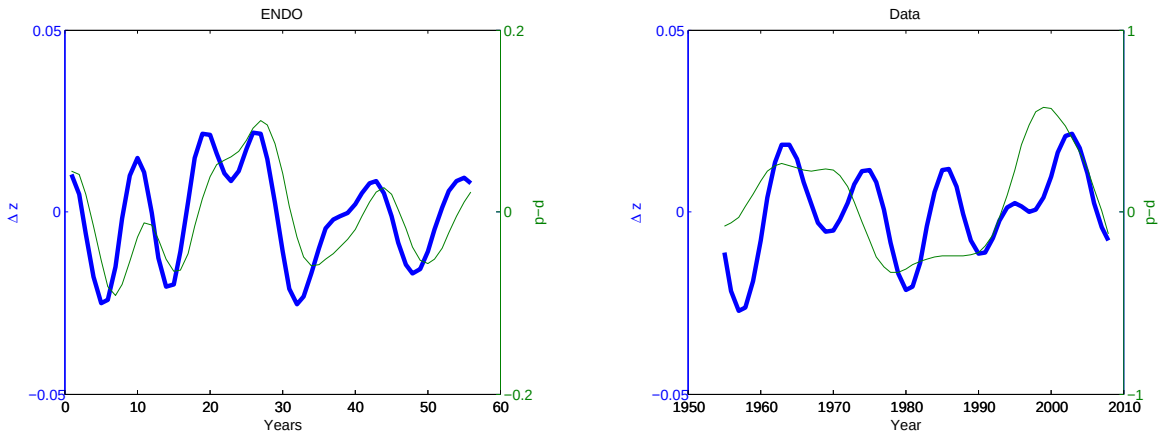
This figure shows quarterly log-deviations from the steady state for the ENDO (solid line) and EXO (dashed line) models. All deviations are multiplied by 100.

Figure 6: Low-Frequency Growth Components



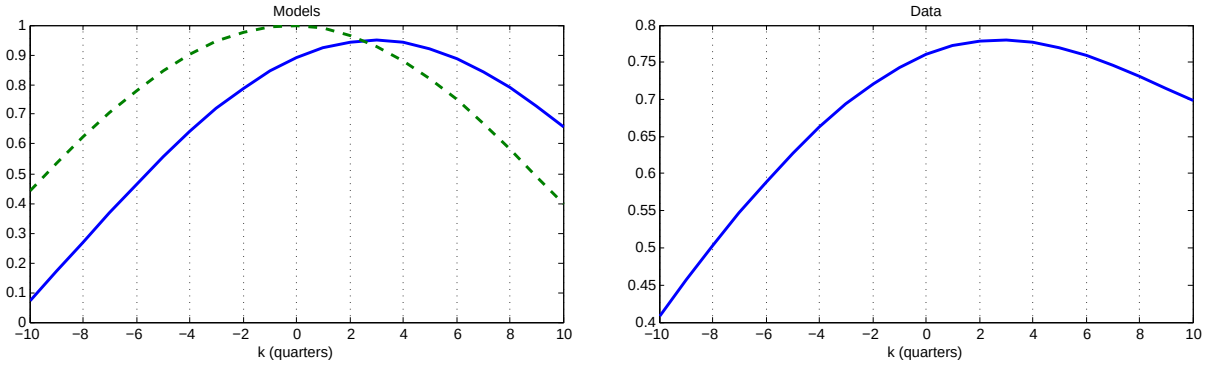
This figure plots the low-frequency growth components for productivity (dashed line), output (thin line), and consumption (bold line). The left panel corresponds to a sample simulation from the ENDO model and the right panel corresponds to the data. The low-frequency component is obtained by applying the bandpass filter from Christiano and Fitzgerald (2003) to annual data and selecting a bandwidth of 32 to 100 quarters. Annual data on GDP and consumption are from the BEA and annual productivity data are from the BLS.

Figure 7: Low-Frequency Component of Productivity Growth and Price-Dividend Ratio



This figure plots the low-frequency components for productivity growth (bold line) and for the price-dividend ratio (thin line). The left panel corresponds to a sample simulation from the ENDO model and the right panel corresponds to the data. The low-frequency component is obtained by applying the bandpass filter from Christiano and Fitzgerald (2003) to annual data and selecting a bandwidth of 32 to 100 quarters. The correlation between the two series is 0.46 in the data and 0.67 in the model. Annual data on productivity are from the BLS and price-dividend data are from CRSP.

Figure 8: Low-Frequency Cross-Correlation of Returns and Consumption Growth



The left panel plots cross-correlations of the medium-frequency component of the equity return and the low-frequency component of consumption growth for the ENDO (bold line) and EXO (dashed line) models: $corr(r_{d,t}, \Delta c_{t+k})$. The right panel plots the same cross-correlations from the data. The low-frequency component is obtained using the bandpass filter from Christiano and Fitzgerald (2003) and selecting a bandwidth of 32 to 100 quarters. Quarterly consumption data is obtained from the BEA. Monthly return data is obtained from CRSP and then compounded to a quarterly frequency.